Economic Ordering Quantity Model for Non-instantaneous Deteriorating Items under Conditions of Permissible Delay in Payments

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Abstract

The study proposes an inventory model for non-instantaneous deteriorating goods over a finite time horizon. In real condition, many goods can be kept for a period without deterioration, but a fixed duration later the goods is starting to deteriorate. We named these kinds of goods as “non-instantaneous deteriorating items”. Under the situation which the demand is increasing linearly with time and allowing delay in payment, the inventory model in this study is divided into four cases by the time of shortage and deadline of delay in payment, and we aim to find the minimum relevant inventory cost per unit time. Numerical examples are given to illustrate the solution procedure.

***Keywords*: Inventory model, Non-instantaneous Deteriorating items, Delay in Payment

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1. INTRODUCTION

The inventory system is taking an important part of cost controlling in business operation. Under the situation of lower profit result from intense competition, how to reduce the cost for enhancing competitiveness is becoming an important issue in business, therefore constructing an optimal inventory model to minimize the cost will determine the business future.

It is necessary for business to investigate the market demand when determining the ordering quantities of goods. Demand rate was considered in a constant in past articles. Donaldson(1977) first derived an inventory model with a linearly increasing trend of demand over a finite horizon. Wee(1995) established an inventory model for declining market that demand rate decrease exponentially with time. Bhunia and Maiti(1999) developed an inventory model with linear increasing demand over a finite planning horizon.

Most goods will deteriorate, spoil or exceeded their expiration date (e.g. corns, fruits or alcohol) during inventory, and they will bring the handling cost for business. Ghare and Schrader(1963) integrated the concept of deteriorating into the inventory model first, in this model the deteriorating rate was assumed as a constant. Abad(2000) extended this concept into EPQ model. However, Wu et al.(2006) indicated that many goods will not deteriorating in beginning, but it would start to deteriorate later in a fixed duration. We named these kinds of goods as “non-instantaneous deteriorating items”. Initializing the model, the deteriorate rate is zero but become a constant later in a fixed period.

In general, Business must pay the purchase cost when receiving goods in general, but most suppliers would provide a credit period for business in real situation. Business would earn interest income during this period, and pay interest charge after the trade credit period delay in opposite. Haley and Higgins(1973) derived an inventory model allows delay in payment first. Goyal(1985) developed an EOQ model including interest earning and charging under the situation permissible delay in payment. Aggarwal and Jaggi(1995) extended Goyal’s model for deteriorating items, and assumed the deteriorating rate as a constant. Jamal et al.(1997) then extended Aggarwal and Jaggi(1995)’s model to allow shortage. Moreover, there are several articles involved in credit transaction in different conditions such as Liao et al.(2000), Chang and Dye(2001), Chang(2004), Liao(2007).

Under the situation which the demand is linear trend with time and allowing delay in payment over a finite horizon, the inventory model in this study is divided into four cases by the time of shortage and deadline of delay in payment. The study aimed to establish an algorithm for finding the minimum relevant inventory cost per unit time, and we assumed the demand rate demand is linear trend with time. Beginning the inventory, the goods wouldn’t deteriorate, but starting to deteriorate in a constant rate after a fixed period later. Finally, numerical examples are given to illustrate the solution procedure.

2. NOTATIONS AND ASSUMPTIONS

2.1 Notations

H	length of planning horizon
T	length of replenishment cycle (H/n)
$I(t)$	the level of inventory at time t
n	the number of replenishment cycles
$D(t)$	demand rate per unit time
θ	deterioration rate, $(0 \leqq \theta \leqq 1)$
M	permissible delay in settling accounts
t_d	length of time with no deterioration $(0 \leqq t_d \leqq T)$
k	length of period with positive inventory $(t_d \leqq k \leqq T)$
I_e	the interest earned per \$ per year
I_c	the interest charged per \$ in stocks per year by the supplier
S	the fixed cost per order
C_h	the holding cost per unit per unit time
C_d	the deterioration cost (\$/ unit)
C_s	the shortage cost per unit per unit time
C_p	unit purchasing price per item (\$/ unit)
P	unit selling price (\$/unit)

2.2 Assumptions

1. A single item is considered over a finite planning horizon.
2. Shortages are allowed except for the last cycle
3. The demand rate $D(t)$ at time t is assumed to be $a+bt$.
4. Initializing the cycle, the goods wouldn't deteriorate, but starting to deteriorate in a constant rate(θ) after a fixed period(t_d) later.
5. During the credit period, business can earn the interest income. After this period, business starts paying for interest charges.

3. MODEL DEVELOPMENT

There are n cycles over a finite planning horizon, and every cycles are beginning at $(j-1)T$ (j=1,2…n). The inventory model begin at t=0 while the inventory level can be present as $Q(1)$. The study assumed that the goods would start to deteriorate in a constant rate after time $(j-1)T+t_d$, and shortage occurs during the period $(j-1)T+k$ to jT (j=1,2…n). We aimed to find the optimal n and k out over the period [0,H] for finding the optimal cost in model.

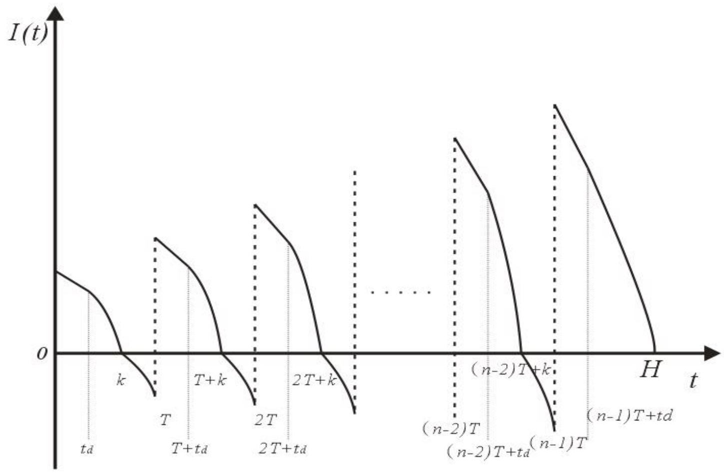

Fig. 1. Inventory System

The change of inventory level can be described by the following equations:

$$\frac{dI_1(t)}{dt} = -(a+bt) \qquad (j\text{-}1)T < t < (j\text{-}1)T + t_d, \qquad \textbf{(1)}$$

$$\frac{dI_2(t)}{dt} + \theta I(t) = -(a+bt) \qquad (j\text{-}1)T + t_d < t < (j\text{-}1)T + k \qquad \textbf{(2)}$$

$$\frac{dI_3(t)}{dt} = -(a+bt) \qquad (j\text{-}1)T + k < t < jT \qquad \textbf{(3)}$$

In addition, from the boundary condition $I((j\text{-}1)T)=I_{max}(j)$ and $I((j\text{-}1)+k)=0$, the solution of (1)-(3) are

$$I_1(t) = Q(j) - \frac{(t-(j-1)T)(2a+b((j-1)T+t))}{2} \qquad (j\text{-}1)T < t < (j\text{-}1)T + t_d \qquad \textbf{(4)}$$

$$I_2(t) = \frac{1}{\theta^2 e^{t\theta}}(e^{t\theta}(b-(a+bt)\theta) + (a\theta + b(((j-1)T+k)\theta - 1))e^{(k+(j-1)T)\theta}) \qquad (j\text{-}1)T + t_d < t < (j\text{-}1)T + k \qquad \textbf{(5)}$$

$$I_3(t) = \frac{(k-t+(j-1)T)(2a+(k+t+(j-1)T)b)}{2} \qquad (j\text{-}1)T + k < t < jT \qquad \textbf{(6)}$$

The states of the inventory level in the last cycle can be described by the following equations (shortage is not allowed in this cycle)

$$I_4(t) = Q(n) - \frac{(t-(j-1)T)(2a+b((j-1)T+t))}{2} \qquad (n\text{-}1)T < t < (n\text{-}1)T + t_d \qquad \textbf{(7)}$$

$$I_5(t) = \frac{e^{t\theta}(b-(a+bt)\theta) + e^{H\theta}(a\theta + (H\theta - 1)b)}{\theta^2 e^{t\theta}} \qquad (n\text{-}1)T + t_d < t < H \qquad \textbf{(8)}$$

We can solve $Q(j)$ from Eqs.(4)(5) while $t=(j\text{-}1)T+t_d$

$$Q(j)=\frac{1}{2\theta^2}(2a\theta(e^{(k-t_d)\theta}+t_d\theta-1)+b(2((j-1)T\theta+k\theta-1)e^{(k-t_d)\theta} \\ +(t_d\theta-2)t_d\theta+2(j-1)T\theta(t_d\theta-1)+2)) \tag{9}$$

similarly,

$$Q(n)=\frac{1}{2\theta^2}(2a\theta(e^{(H-t_d-(n-1)T)\theta}+t_d\theta-1)+b(H\theta-1)e^{(H-t_d-(n-1)T)\theta} \\ +(t_d\theta-2)t_d\theta+2(n-1)T\theta(t_d\theta-1)+2)) \tag{10}$$

The holding cost (HC) is given by

$$HC=C_h\left(\sum_{j=1}^{n-1}\left(\int_{(j-1)T}^{(j-1)T+t_d}I_1(t)dt+\int_{(j-1)T+t_d}^{(j-1)T+k}I_2(t)dt\right)+\int_{(n-1)T}^{(n-1)T+t_d}I_4(t)dt+\int_{(n-1)T+t_d}^{H}I_5(t)dt\right) \tag{11}$$

The deterioration cost (DC) is given by

$$DC=C_d\left(\sum_{j=1}^{n-1}\left(Q(j)-\int_{(j-1)T}^{(j-1)T+k}(ae^{-bt})dt\right)+Q(n)-\int_{(j-1)T}^{H}(ae^{-bt})dt\right) \tag{12}$$

The shortage cost (SC) is given by

$$SC=-C_s\sum_{j=1}^{n-1}\int_{(j-1)T+k}^{jT}I_3(t)dt \tag{13}$$

We divide the model into four cases by the time of shortage and deadline of delay in payment for computing interest revenue and expenditure:

Case 1 $0\le M\le t_d$

The length of delay in payment(M) is absolutely less than the length with no deterioration(t_d) in this case. Business can earn the interest income during the credit period. However, business starts paying for interest charges after the credit period.

Total interest expenditure is given by

$$C_pI_c\left(\sum_{j=1}^{n-1}\left(\int_{(j-1)T+M}^{(j-1)T+t_d}I_1(t)dt+\int_{(j-1)T+t_d}^{(j-1)T+k}I_2(t)dt\right)+\int_{(n-1)T+M}^{(n-1)T+t_d}I_4(t)dt+\int_{(n-1)T+t_d}^{H}I_5(t)dt\right) \tag{14}$$

Total interest revenue is given by

$$PI_e\sum_{j=1}^{n}\int_{(j-1)T}^{(j-1)T+M}(a+bt)(t-(j-1)T)dt \tag{15}$$

Thus, we have

$$
\begin{aligned}
TC_1 = nS &+ C_h\left(\sum_{j=1}^{n-1}\left(\int_{(j-1)T}^{(j-1)T+t_d} I_1(t)dt + \int_{(j-1)T+t_d}^{(j-1)T+k} I_2(t)dt\right) + \int_{(n-1)T}^{(n-1)T+t_d} I_4(t)dt + \int_{(n-1)T+t_d}^{H} I_5(t)dt\right) \\
&+ C_d\left(\sum_{j=1}^{n-1}\left(Q(j) - \int_{(j-1)T}^{(j-1)T+k}(a+bt)dt\right) + Q(n) - \int_{(j-1)T}^{H}(a+bt)dt\right) - C_s\sum_{j=1}^{n-1}\int_{(j-1)T+k}^{jT} I_3(t)dt \\
&+ C_p I_c\left(\sum_{j=1}^{n-1}\left(\int_{(j-1)T+M}^{(j-1)T+t_d} I_1(t)dt + \int_{(j-1)T+t_d}^{(j-1)T+k} I_2(t)dt\right) + \int_{(n-1)T+M}^{(n-1)T+t_d} I_4(t)dt + \int_{(n-1)T+t_d}^{H} I_5(t)dt\right) \\
&- PI_e\sum_{j=1}^{n}\int_{(j-1)T}^{(j-1)T+M}(a+bt)(t-(j-1)T)dt
\end{aligned}
\tag{16}
$$

Case 2 $t_d \le M \le k$

The main difference to Case 1 is that the period of delay in payment(*M*) is more length than the period with no deterioration(t_d) but less than length of period with positive inventory(*k*).

Total interest expenditure is given by

$$
C_p I_c\left(\sum_{j=1}^{n-1}\int_{(j-1)T+M}^{(j-1)T+k} I_2(t)dt + \int_{(n-1)T+M}^{H} I_5(t)dt\right) \tag{17}
$$

Total interest revenue is given by

$$
PI_e\sum_{j=1}^{n}\int_{(j-1)T}^{(j-1)T+M}(a+bt)(t-(j-1)T)dt \tag{18}
$$

Thus, we have

$$
\begin{aligned}
TC_2 = nS &+ C_h\left(\sum_{j=1}^{n-1}\left(\int_{(j-1)T}^{(j-1)T+t_d} I_1(t)dt + \int_{(j-1)T+t_d}^{(j-1)T+k} I_2(t)dt\right) + \int_{(n-1)T}^{(n-1)T+t_d} I_4(t)dt + \int_{(n-1)T+t_d}^{H} I_5(t)dt\right) \\
&+ C_d\left(\sum_{j=1}^{n-1}\left(Q(j) - \int_{(j-1)T}^{(j-1)T+k}(a+bt)dt\right) + Q(n) - \int_{(j-1)T}^{H}(a+bt)dt\right) - C_s\sum_{j=1}^{n-1}\int_{(j-1)T+k}^{jT} I_3(t)dt \\
&+ C_p I_c\left(\sum_{j=1}^{n-1}\int_{(j-1)T+M}^{(j-1)T+k} I_2(t)dt + \int_{(n-1)T+M}^{H} I_5(t)dt\right) - PI_e\sum_{j=1}^{n}\int_{(j-1)T}^{(j-1)T+M}(a+bt)(t-(j-1)T)dt
\end{aligned}
\tag{19}
$$

Case 3 $k \le M \le T$

In case 3, the period of delay in payment(*M*) is more length than period with positive inventory(*k*) except for the last cycle. Therefore, the interest charge occurs in the last cycle only.

Total interest expenditure is given by

$$
I_c C_p \int_{(n-1)T+M}^{H} I_5(t)dt \tag{20}
$$

Total interest revenue is given by

$$PI_e(\sum_{j=1}^{n}\left(\int_{(j-1)T}^{(j-1)T+k}(a+bt)(t-(j-1)T)dt+(M-k)\int_{(j-1)T}^{(j-1)T+k}(a+bt)dt\right) + \int_{(n-1)T}^{(n-1)T+M}(a+bt)(t-(n-1)T)dt) \tag{21}$$

Thus, we have

$$\begin{aligned} TC_3 = nS &+ C_h\left(\sum_{j=1}^{n-1}\left(\int_{(j-1)T}^{(j-1)T+t_d} I_1(t)dt+\int_{(j-1)T+t_d}^{(j-1)T+k} I_2(t)dt\right)+\int_{(n-1)T}^{(n-1)T+t_d} I_4(t)dt+\int_{(n-1)T+t_d}^{H} I_5(t)dt\right) \\ &+ C_d\left(\sum_{j=1}^{n-1}\left(Q(j)-\int_{(j-1)T}^{(j-1)T+k}(a+bt)dt\right)+Q(n)-\int_{(j-1)T}^{H}(a+bt)dt\right)-C_s\sum_{j=1}^{n-1}\int_{(j-1)T+k}^{jT} I_3(t)dt \\ &+ I_cC_p\int_{(n-1)T+M}^{H} I_5(t)dt - PI_e\sum_{j=1}^{n}\left(\int_{(j-1)T}^{(j-1)T+k}(a+bt)(t-(j-1)T))dt\right. \\ &\left.+(M-k)\int_{(j-1)T}^{(j-1)T+k}(a+bt)dt\right)-PI_e\int_{(n-1)T}^{(n-1)T+M}(a+bt)(t-(n-1)T)dt \end{aligned} \tag{22}$$

Case 4 $T \le M$

The period of delay in payment(M) is absolutely more length than a cycle(T) in Case 4. To compare with Case 3, there is no interest charge in this case.

Total interest revenue is given by

$$PI_e\left(\sum_{j=1}^{n}\left(\int_{(j-1)T}^{(j-1)T+k}(a+bt)(t-(j-1)T)dt+(M-k)\int_{(j-1)T}^{(j-1)T+k}(a+bt)dt\right) + \int_{(n-1)T}^{H}(a+bt)(t-(n-1)T)dt+(M-T)\int_{(n-1)T}^{H}(a+bt)dt\right) \tag{23}$$

Thus, we have

$$\begin{aligned} TC_4 = nS &+ C_h\left(\sum_{j=1}^{n-1}\left(\int_{(j-1)T}^{(j-1)T+t_d} I_1(t)dt+\int_{(j-1)T+t_d}^{(j-1)T+k} I_2(t)dt\right)+\int_{(n-1)T}^{(n-1)T+t_d} I_4(t)dt+\int_{(n-1)T+t_d}^{H} I_5(t)dt\right) \\ &+ C_d\left(\sum_{j=1}^{n-1}\left(Q(j)-\int_{(j-1)T}^{(j-1)T+k}(a+bt)dt\right)+Q(n)-\int_{(j-1)T}^{H}(a+bt)dt\right)-C_s\sum_{j=1}^{n-1}\int_{(j-1)T+k}^{jT} I_3(t)dt \\ &- PI_e\left(\sum_{j=1}^{n}\left(\int_{(j-1)T}^{(j-1)T+k}(a+bt)(t-(j-1)T)dt+(M-k)\int_{(j-1)T}^{(j-1)T+k}(a+bt)dt\right)\right. \\ &\left.+\int_{(n-1)T}^{H}(a+bt)(t-(n-1)T)dt+(M-T)\int_{(n-1)T}^{H}(a+bt)dt\right) \end{aligned} \tag{24}$$

While t_d=M, the result of TC_1 is equal to TC_2. Similar, while k=M, the result of TC_2 is equal to TC_3, and while T=M, the result of TC_3 is equal to TC_4.

4. SPECIAL CASE

In this study, there are several special cases under different conditions:

(1) When t_d=0 and M=0, the model is similar to Bhunia and Maiti(1995).

(2) When t_d=0 and b=0, the model is similar to Chang et al. (2002).

5. ALGORITHM

1. If $M<t_d$, the optimal solution will situate in Case 1, then we can calculate the total cost from Eqs. (16), otherwise the optimal solution may situate around Case 2 to 4, and we calculate the total cost from Eqs. (19)(22)(24), individually.
2. Note that both n and k are decision variables in the study, and n is integer. Consequently, Increasing n by 1 until $TC_i(n,k^*)$ stop reducing. Take the first and second derivatives of Eqs.(16)(19)(22)(24) with respect to k, and checking the Eqs.(25)(26) are satisfied.

$$\frac{\partial TC_i}{\partial k}=0 \tag{25}$$

$$\frac{\partial^2 TC_i}{\partial k^2}>0 \tag{26}$$

3. If $M>t_d$, we must compare the optimal cost from Case 1 to Case 3, and find the minimum $TC_i(n^*,k^*)$ as optimal case.

6. NUMERICAL EXAMPLES

In order to illustrate the solution procedure, we consider the following data a=3000, b=1800, θ=0.1, H=1, t_d=15/365, S=100, C_h=3, C_d=7, C_s=6, C_p=8, P=12, I_e=0.03, I_c=0.08, and we have two examples of M=10/365(0.027397 year) and 30/365(0.082192 year).

Example 1

In the Example 1, M=10/365. To compare between M and t_d, we can find $M<t_d$. Through the algorithm, we calculate the total cost from Eqs. (16), and computed results are presented in Table 1. The optimal number of cycles is 8 which the optimal total cost is $1220.06 and k^*=0.08727.

Table 1 Numerical example *M*=10/365

	Case 1	
n	*TC(n,k)*	*k*
6	1437.81	0.10104
7	1392.12	0.08727
8*	1386.79*	0.07694*
9	1407.37	0.06890
10	1445.56	0.06247

Example 2

In the Example 2, M=30/365. To compare between M and t_d, we can find $M>t_d$. Through the algorithm, we calculate the total cost by Eqs. (19)(22)(24), and computed results are presented in Table 2 individually.

In case 2, the optimal number of cycles is 8 which the minimum total cost is \$1319.72 and $k^*=M$.

In case 3, the optimal number of cycles is 8 which the minimum total cost is \$1319.25 and $k^*=0.08033$.

In case 4, the optimal number of cycles is 13 which the minimum total cost is \$1565.07 and $k^*=0.05169$.

To compare the optimal cost among cases, we obtain when n=8 and k=0.08033, the minimum total cost will be \$1319.72 in case 3.

Table 2 Numerical example *M*=30/365

Case 2			Case 3			Case 4		
n	*TC(n,k)*	*k*	*n*	*TC(n,k)*	*k*	*n*	*TC(n,k)*	*k*
6	1362.77	0.10443	6	1411.03	*k=M*	12	-	
7	1321.02	0.09066	7	1329.49	*k=M*	13*	1565.07*	0.05169*
8*	1319.72*	*k=M**	8*	1319.25*	0.08033*	14	1638.08	0.04841
9	1358.73	*k=M*	9	1342.83	0.07206	15	1715.11	0.04557
10	1432.43	*k=M*	10	1383.35	0.06544	16	1795.38	0.04309

7. SENSITIVITY ANALYSIS

In this section, we use the same numerical example. The study aimed to analyze the influence of the parameters with respect to n, k and TC. The sensitivity analysis results are presented in table 3.

Table 3 Sensitivity analysis

Parameters	Change(%)	The Optimal Case	TC^*	k^*	n^*
a	40	Case 3	1475.69	0.080318	8
	20	Case 3	1397.47	0.080324	8
	-20	Case 2	1227.73	0.090662	7
	-40	Case 2	1134.41	0.104432	6
b	40	Case 3	1370.51	0.080347	8

Parameters	Change(%)	The Optimal Case	TC^*	k^*	n^*
	20	Case 3	1344.88	0.080340	8
	-20	Case 2	1290.10	0.090662	7
	-40	Case 2	1259.19	0.090662	7
M	40	Case 3	1285.92	0.081506	8
	20	Case 3	1302.46	0.080919	8
	-20	Case 2	1337.17	0.079313	8
	-40	Case 2	1357.10	0.078296	8
t_d	40	Case 2	1304.65	0.091800	7
	20	Case 3	1311.51	0.080916	8
	-20	Case 3	1328.36	0.079749	8
	-40	Case 3	1338.83	0.079168	8

From Table 3, the following points are presented:

(1)The total cost(TC) is increasing(decreasing) with increase(decrease) the value of parameters a and b.

(2)The total cost(TC) is increasing(decreasing) with decrease (increase) the value of parameters M and t_d, but the total cost(TC) is lowly sensitive to changes in t_d.

(3) The decision variable k is lowly sensitive to changes in parameters a, b, M and t_d for any fixed n.

8. CONCLUSION

In this paper, we present an inventory model for non-instantaneous deteriorating items with permissible delay in payment over a finite horizon. The study provided an algorithm for business to find the optimal inventory strategy. Finally, sensitivity analysis is given to present the effect of changes in the system parameters a,b,M and t_d on optimal total cost. Through the sensitivity analysis, business could clearly understand what factor is more important in inventory system. Moreover, the proposed model can be extended in several situations. First, the study assumed goods will start to deteriorate in a constant rate after a fixed period later. We could extend the constant deterioration rate to Weibull distribution or exponential distribution. Second, we could also generalize the model to allow for partial backlogging. Third, the proposed model could consider the time-value of money further.

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