製程能力弱度指標 C_{pp} 在分群樣本下的實務應用

Practical Implementation of the C_{pp} Index when Using Subsamples

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Abstract

Capability indices are key measures in the context of never-ending improvement in quality. Confidence bounds are derived for the common measures of process capability. The process measures are estimated based on a single random sample of observations from the normally distributed process, which is in statistical control. In practice, and in much of the quality control literature, process data are collected over time in subsamples representing rational subgroups. In this paper, therefore, we use the Patnaik's (1950) approximation to construct the estimation and capability testing of C_{pp} based on multiple samples. An example is also given to demonstrate this simple approximate procedure for judging whether a stable process meets the present capability requirement.

***Keywords*:** Capability Indices, Confidence Bound, Critical Value, *p*-value

1. Introduction

Capability analysis is an important step in implementing a control system. And, for a current summary of advancements and requirements in the field of capability analysis, the interested reader is directed to the review paper with discussion, Kotz and Johnson (2002) issue of the Journal of Quality Technology. In practice, many quality characteristics can be expressed in terms of a numerical measurement when dealing with a quality characteristic that is a variable. Usually it needs to monitor both the mean value of the quality characteristic and its variability. Control of the process mean quality level is usually with the control chart for the $\overline{X}$ chart. Process variability can be monitored with either a control chart for the *S* chart or the *R* chart.

Suppose that a quality characteristic is normally distributed with mean μ and standard deviation σ. If $X_1, X_2, \ldots, X_n$ is a sample of size *n*, then the mean of this sample is $\overline{X}$. In most cases, both μ and σ are unknown. Therefore, they need to be estimated from prelim -inary samples or subgroups of sample from process, which is in control. These estimates usually are based on at least 20 to 25 samples. Suppose that *m* subsamples are available and each sample contains *n* observations on the quality characteristic. Typically, *n* will be small, often 4, 5, or 6.

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Let $\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_m$ be the average of each subsamples then a reasonable estimator of μ, the process mean. is given by

$$\overline{\overline{X}} = (\overline{X}_1 + \overline{X}_2 + \cdots + \overline{X}_m)/m. \tag{1.1}$$

To construct the estimation and capability testing of C_{pp} based on multiple samples, we need an estimate of the standard deviation σ. We may estimate σ either by the sample standard deviation or the range of the *m* subsample means. For the present, we concentrate on the range method. If $X_1, X_2, \ldots, X_n$ is a sample of size *n*, then the range of the sample is the difference between the largest and smallest observations; that is

$$R = X_{max} - X_{min} \tag{1.2}$$

where X_{max}= max{ $X_1, X_2, \ldots, X_n$ } and X_{min}= min{ $X_1, X_2, \ldots, X_n$ }. There is a well-know relationship between the range of a sample from a normal distribution and the standard deviation of that distribution. The random variable $Y = R/\sigma$ is called the relative range. The parameters of the distribution of *Y* are a function of the sample size *n*. If the expected value of the *Y* is d_2, then $E(R/d_2)$ = σ. Consequently, an estimator of σ is $\hat{\sigma} = R/d_2$. Therefore, if $\overline{R}$ is the average range of the *m* preliminary samples, we may use $\hat{\sigma} = \overline{R}/d_2$ to estimate σ.

Ott (1975) points out, if the sample size is relatively small, the range estimate yields a good an estimator of the variance σ^2 based on a single sample. For moderate value of *n*, say, $n \geq 10$, the range method loses its efficiency rapidly, as it ignores all the information in the sample between X_{max} and X_{min}. However, for the small sample size, *n* = 4, 5, or 6, often employed on variables control charts, it is entirely satisfactory.

For thorough discussions of different capability indices, e.g., the firstly proposed process capability indices are C_p and C_{pk} which were developed by Kane(1986). Boyles(1991) pointed out the C_p and C_{pk} are yield-based indices, which are independent of the target *T*, which fail to account for process centering. Chan et al. (1988) developed the index C_{pm} in order to take into account the process centering being defined as follows

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}. \tag{1.3}$$

Greenwich and Jahr-Schaffrath (1995) introduced a new index C_{pp} which is easier to use and analytically convenient. The index C_{pp} is defined as follows

$$C_{pp} = \frac{1}{C_{pm}^2} = \frac{(\mu - T)^2}{D^2} + \frac{\sigma^2}{D^2}, \tag{1.4}$$

where $D = min\{USL - T, T - LSL\} = d/3$, $[LSL, USL]$ is the specification interval, μ is the process mean, σ is the process standard deviation(overall process variability), $d = (USL - LSL)/2$ is half the length of the specification interval, and *T* is the target value. Let $(\mu - T)^2/D^2$ be denoted by C_{ia} (inaccuracy index) and σ^2/D^2 be denoted by C_{ip} (imprecision index). Thus, $C_{pp} = C_{ia} + C_{ip}$ and it is a simple transformation of the index C_{pm}, under stationary controlled conditions.

Owing to the relation C_{pp}, which assumes a smaller value for a process more capable meeting its specifications and a larger value for a less capable process. Any non-zero value of C_{pp} indicates some degree of incapability of the process. And these sub-indices also are providing the proportions of the process incapability contributed by the departure of the process mean from the target and the process variation, respectively.

When using subsamples, Li et al.(1990) have studied the distribution of the estimators of C_p and C_{pk} base on ranges. Kirmani et al.(1991) have studied the distribution of the estimators of C_p base on sample standard deviations of the subsamples.

In this article, therefore, we will focus on the estimators of the process parameter σ by the ratios of sample range divided by d_2. We, besides, apply the derived distribution to study the use of hypothesis testing to assess process capability. Also, we give the tables of the confidence bounds and *p*-value on the capability indices based on range, justifying whether the process potential and performance to meeting consumer's expectation specification.

2. Approximate confidence bound for C_{pp} based on range

In this section, we derive confidence intervals for C_{pp}, based on range. Denote by $X_1, X_2, \ldots, X_n$ a random sample of *n* observations, drawn from a normal population having mean μ and standard deviation σ. Then the range in this sample is denoted by $R = X_{max} - X_{min}$.

Suppose the total sample are grouped to *m* subsamples such that each subsample contain *n* observations, the mean of the *m* ranges will then be denoted by $\overline{R}_{m,n}$ and $R_{1,n}$ is the range of a sample of size *n*.

Let $E(R) = \sigma d_2$ and $Var(R) = \sigma^2 d_3^2$. Then the mean and variance of the distribution of $\overline{R}_{m,n}/\sigma$ are given by, respectively,

$$E(\overline{R}_{m,n}/\sigma) = E(R_{1,n}/\sigma) = d_2, \tag{2.1}$$

and

$$Var(\overline{R}_{m,n}/\sigma) = Var(R_{1,n}/\sigma)/m = d_3^2/m. \tag{2.2}$$

Then $\overline{R}_{m,n}/d_2$ is an unbiased estimate of σ, where d_2 and d_3 are constants see Hartley and Pearson (1951). According to Patnaik(1950) it has been shown that $\overline{R}_{m,n}/\sigma$ is distributed approximately as $c\chi/\sqrt{\nu}$. Thus,

$$(\overline{R}_{m,n}/\sigma)^2 \times \nu/c^2 \approx \chi_\nu^2, \tag{2.3}$$

where χ_ν^2 has the chi-square distribution with ν degrees of freedom and *c* and ν are constants which are function of the first two moments of the range. Using these relations, we can easily obtain the values of c and ν for any n and m. We refer the results based on the Patnaik's approximation to "approximate results", where $\nu = 1/(-2+2\sqrt{1+2(d_3/d_2)^2/m})$ and $c = d_2 \times \sqrt{\nu/2} \times \Gamma(\nu/2)/\Gamma((\nu+1)/2) \approx d_2(1+1/(4\nu))$.

Assume that the process measurement follows $N(\mu, \sigma^2)$, the normal distribution, the index and reasonable estimators of C_{pp}, as following

$$\hat{C}_{pp} = \frac{(\overline{\overline{X}} - T)^2}{D^2} + \frac{\hat{\sigma}^2}{D^2}, \quad \hat{\sigma}^2 = \sum_{i=1}^{n}(X_i - T)^2 \Big/ (n-1). \tag{2.4}$$

From (1.4) and (2.4), we have

$$\hat{\sigma}^2/\sigma^2 = (n-1)(1+\lambda/n)/(n-1+\hat{\lambda}) \times \hat{C}_{pp}/C_{pp}. \tag{2.5}$$

Since $\hat{\sigma} = R'/d_2$ is an unbiased estimate of σ, where R' indicates either $\overline{R}_{m,\,n}$ or $R_{1,\,n}$. (That is, either the mean range of m samples or the range for a single sample of size n.) We obtain that

$$(n-1)(1+\lambda/n)/(n-1+\hat{\lambda}) \times \hat{C}_{pp}/C_{pp} \times d_2^2\,\nu/c^2 = (R'/\sigma)^2 \times \nu/c^2 \approx \chi_\nu^2. \tag{2.6}$$

where $\lambda = n(\mu - T)^2/\sigma^2$ is unknown non-central chi-square distribution parameter. Its reasonable estimate $\hat{\lambda}$ is defined by $\hat{\lambda} = n(\overline{\overline{X}} - T)^2/\hat{\sigma}^2$ based on range.

Now, let $w_{pp} = w_{pp}(X_1, \cdots, X_n)$ be a statistics satisfying

$$1-\alpha = P(C_{pp} \le w_{pp})$$

$$= P\left(\frac{\nu}{c^2} \times \frac{R'^2}{\sigma^2} \ge \frac{d_2^2\,\nu}{c^2} \times \frac{(n-1)(1+\lambda/n)}{n-1+\hat{\lambda}} \times \frac{\hat{C}_{pp}}{w_{pp}}\right), \tag{2.7}$$

where $1-\alpha$ does not depend on C_{pp}. We have

$$d_2^2\,\nu/c^2 \times (n-1)(1+\lambda/n)/(n-1+\hat{\lambda}) \times \hat{C}_{pp}/w_{pp} = \chi_{1-\alpha}^2(\nu). \tag{2.8}$$

Thus the $100(1-\alpha)\%$ approximate upper confidence bounds on $C_{pp}/\hat{C}_{pp}$ based on n and $\hat{\lambda}$ is

$$w_{pp}/\hat{C}_{pp} = \frac{(n-1)(1+\hat{\lambda}/n)}{n-1+\hat{\lambda}} \times \frac{d_2^2\,\nu}{c^2} \times \frac{1}{\chi_{1-\alpha}^2(\nu)}$$

$$=2\left(\frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)}\right)^2\times\frac{(n-1)(1+\hat{\lambda}/n)}{n-1+\hat{\lambda}}\times\frac{1}{\chi^2_{1-\alpha}(\nu)}, \quad (2.9)$$

where $\chi^2_{1-\alpha}(\nu)$ is the upper $100(1-\alpha)$ % of the Chi-square distribution with ν degrees of freedom. At the juncture, the ν degrees of freedom for the chi-square distribution may be integer or not integer then we may approximate chi-square by interpolating value of chi-square value.

Tables 1 ~ 2 tabulate the $100(1-\alpha)$ % approximate upper confidence limits for $C_{pp}/\hat{C}_{pp}$, when n and $\hat{\lambda}$ are given. For $\hat{\lambda}=0$ and $\hat{\lambda}=1$, the approximate upper confidence bounds on $C_{pp}/\hat{C}_{pp}$, it is seen that $C_{pp}/\hat{C}_{pp}$ decreases as n and/or m are increases, for any α. Furthermore, for those $\hat{\lambda}>1$, for fixed m, most of the upper confidence bounds on $C_{pp}/\hat{C}_{pp}$ have a concave downward slightly variation as n increase, for α = 0.05, 0.025, and 0.01.

A process is called capable if the C_{pp} less some prefixed value, say $c_0 = w_{pp}$, value of c_0 may be 1, 0.75, 0.6, 0.5, etc. In our formulation, if

$$\hat{C}_{pp}\leqq\frac{c_0}{2}\times\left(\frac{\Gamma(\nu/2)}{\Gamma((\nu+1)/2)}\right)^2\times\frac{n-1+\hat{\lambda}}{(n-1)\times(1+\hat{\lambda}/n)}\times\chi^2_{1-\alpha}(\nu), \quad (2.10)$$

then we claim that the process is capable at least $100(1-\alpha)$ % of the time.

In special case, we consider $\mu=T$, then $\lambda=0$, and

$$d_2^2\nu/c^2\times\hat{C}_{pp}/C_{pp}=\nu/c^2\times R'^2/\sigma^2\approx\chi^2_\nu. \quad (2.11)$$

Thus, the $100(1-\alpha)$ % approximate upper confidence bounds of the ratio of $C_{pp}/\hat{C}_{pp}$ is given by

$$2\left(\frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)}\right)^2\times\frac{1}{\chi^2_{1-\alpha}(\nu)}. \quad (2.12)$$

3. Test hypothesis

A practice that is becoming increasingly common in industry is to require a supplier to demonstrate process capability as parts of the contractual agreement. Thus, it is frequently necessary to demonstrate that the process capability index meets or less some particular target value, say c_0. The process meets the capability requirement if $C_{pp}\leqq c_0$, and fails to meet the capability requirement if $C_{pp}>c_0$, then we have chosen the usually used benchmark values, c_0, say, 1, 0.75, 0.6 or 0.5, of C_{pp} in quality condition. This is a simple decision-making procedure.

In statistics, it is to test $H_0: C_{pp} > c_0$

$$H_1: C_{pp} \leqq c_0 \tag{3.1}$$

Also, we are using the estimate of C_{pp}, $\hat{C}_{pp}$, as the statistic and will evaluate p-value to make a decision. The p-value is the probability of wrongly concluding that an incapable process is capable. First, when $C_{pp} = c_0$, then the critical value c_{pp}/c_0 is determined by

$$\alpha = P(\hat{C}_{pp} < c_{pp} \mid C_{pp} = c_0)$$

$$= P\left(\chi_v^2 < \frac{d_2^2 v}{c^2} \times \frac{(n-1)(1+\lambda/n)}{n-1+\hat{\lambda}} \times \frac{c_{pp}}{c_0} \middle| C_{pp} = c_0 \right), \tag{3.2}$$

where α is given significant level. Hence, we have

$$\frac{d_2^2 v}{c^2} \times \frac{(n-1)(1+\lambda/n)}{n-1+\hat{\lambda}} \times \frac{c_{pp}}{c_0} = \chi_{1-\alpha}^2(v), \tag{3.3}$$

where $\chi_{1-\alpha}^2(v)$ is the upper $(1-\alpha)$ percentile of the chi-square distribution, with ν degrees of freedom, then we obtain the maximum critical value c_{pp}/c_0 is as following:

$$\left(\frac{\Gamma(v/2)}{\Gamma((v+1)/2)} \right)^2 \times \frac{n-1+\hat{\lambda}}{(n-1)\times(1+\hat{\lambda}/n)} \times \frac{\chi_{1-\alpha}^2(v)}{2}. \tag{3.4}$$

Table 3 ~ 4 gives the $100(1-\alpha)\%$ approximate maximum critical value for c_{pp}/c_0, when $\hat{\lambda}$, α, m, n are given. We find that, for $\hat{\lambda} = 0$ and $\hat{\lambda} = 1$, the approximate maximum critical value for c_{pp}/c_0, it is seen that c_{pp}/c_0 increases as n and/or m are increases, for any α. Moreover, for those $\hat{\lambda} > 1$ and given m, most of the maximum critical value for c_{pp}/c_0 have a slightly concave upward variation as n increase, for α = 0.05, 0.025, and 0.01.

And, $$p\text{-value} = P(\hat{C}_{pp} \le \hat{c}_{pp}) = P\left(\chi_v^2 < \frac{d_2^2 v}{c^2} \times \frac{(n-1)(1+\lambda/n)}{n-1+\hat{\lambda}} \times \frac{\hat{c}_{pp}}{c_0} \middle| C_{pp} = c_0 \right)$$

$$= P\left(\chi_v^2 < 2 \times \left(\frac{\Gamma((v+1)/2)}{\Gamma(v/2)} \right)^2 \times \frac{(n-1)(1+\lambda/n)}{n-1+\hat{\lambda}} \times W_{pp} \middle| C_{pp} = c_0 \right). \tag{3.5}$$

Where $\hat{c}_{pp} = \hat{C}_{pp}$ denotes the observed value of the test statistic, and $W_{pp} = \hat{c}_{pp}/c_0$. Table 5 ~ 8 given the $100(1-\alpha)\%$ approximate p-value for C_{pp}, when $\hat{\lambda}$, m, n are given.

For $W = 0.9$, we find that, p-value of C_{pp} concave downward as n and/or m are increase for $\hat{\lambda} \neq 0$. But, for $W = 1$, for p-value of C_{pp} increases as $\hat{\lambda} \neq 0$ and n increase, but it is decrease, as $\hat{\lambda} = 0$ and n increase. Moreover, for those $W < 1$, p-value of C_{pp} decreases as n and/or m are increase for various $\hat{\lambda}$.

In special case, we consider μ = T, then $\lambda = 0$, and

$$d_2^2 v / c^2 \times C_{pm}^2 / \hat{C}_{pm}^2 = v / c^2 \times (R'/\sigma)^2 \approx \chi_v^2. \tag{3.6}$$

Thus, the maximum critical value c_{pp}/c_0 is $\left(\dfrac{\Gamma(\nu/2)}{\Gamma((\nu+1)/2)}\right)^2 \times \dfrac{\chi^2_{1-\alpha}(\nu)}{2}$.

and the p - value $= P(\hat{C}_{pp} \le \hat{c}_{pp})$

$$= P\left(\chi^2_\nu < 2\times\left(\frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)}\right)^2 \times W_{pp} \middle| C_{pp} = c_0\right). \qquad (3.7)$$

4. The procedure

As stated before, to check it the process meets the capability requirement, we first determine process capability value, c_0, of C_{pp}, and the α-risk. Second, we calculate the estimated value $\hat{C}_{pp}$ from the sample. Third, from the appropriate Table we find the maximum value $\hat{C}_{pp}/c_0$ based on α, $\hat{\lambda}$, m and n. Finally, if the lower confidence bounds is greater than c_0 and the p-value≦α, then we conclude that the process is capable. Otherwise, we do not have sufficient information to conclude that the process meets the present capability requirement. In sum, we summarize these steps shown in Table 9.

Table 9: The step of the *U.C.B.* for $C_{pp}/\hat{C}_{pp}$ and p-value for C_{pp}.

Step C_{pp}
1. Determine the value of c_0 (set to 1, 0.75, 0.6 or 0.5) and α.
2. a. Compute $\hat{\sigma} = \overline{R}_{m,n}/d_2$. b. Calculate the value $\hat{\lambda} = n(\overline{\overline{X}} - T)^2/\hat{\sigma}^2$. c. Calculate $\hat{C}_{pp}$ and $W_{pp} = \hat{C}_{pp}/c_0 = \hat{c}_{pp}/c_0$.
3. a. Find the corresponding *U.C.B.* based on α, $\hat{\lambda}$, m and n. b. Find the corresponding p-value and maximum critical $\hat{C}_{pp}/c_0$ value based on W_{pp}, $\hat{\lambda}$, m and n.
4. a. If the $\hat{C}_{pp}$ times the tabulated *U.C.B.*≦ c_0, conclude that the process is capable; Otherwise, we do not have enough information to conclude that the process is capable. b. If the p-value≦α, conclude that the process is capable; Otherwise, we do not have enough information to conclude that the process is capable. c. If $\hat{C}_{pp}$ is less than c_0 times the tabulated maxi. critical value based on α, $\hat{\lambda}$, m and n, conclude that the process is capable; Otherwise, we do not claim the process is capable.

5. Numerical example

We use the data given in Table 5-1 of Montgomery (2001) to demonstrate this approximate procedure. This example is about a manufacturing process, which produces piston rings for an automotive engine. The measurements are the inside diameter of the rings manufactured in this process with 25 subsamples, each subsample of 5 sample. The *USL* = *74.05* and *LSL* = 73.95, and *T* =*74* = *M*, *m* = 25, *n* = 5. From the process data, we obtain sample mean $\overline{\overline{X}}$ = 74.001176 and the mean of the *m* ranges is denoted by $\overline{R}_{m,\,n}$ = 0.02276, d_2 = 2.32593 and $\hat{\sigma} = \overline{R}_{m,\,n}/d_2$ = 0.009785, and $\hat{\lambda}$ = 0.072216, then $\hat{C}_{pp}$ = 0.349665.
As stated before, to check whether the process was "satisfactory", i.e., $C_{pp} < 0.75$, α = 0.05.

Step 1. Define $C_{pp} \leqq 0.75$, $c_0 = 0.75$.

Step 2. Calculate $\overline{\overline{X}}$ =74.001176, $\overline{R}_{25,\,5}$ = 0.02276, $\hat{\sigma}$ = 0.009785, and

$\hat{\lambda}$ = 0.072216, $\hat{C}_{pp}$ = 0.349665, and W_{pp} = 0.349665/0.75 = 0.46622.

Step 3. From the Tabulate, we obtains

a. Upper confidence limits of C_{pp} is 0.349665×1.288578 =0.450571

b. Maximum critical value of $\hat{C}_{pp}$ is 0.75×0.776049 =0.582037

c. *p*-value(C_{pp}) = 0.000003.

Step 4. For C_{pp}, we can conclude that the process is "satisfactory" because $\hat{C}_{pp}$ = 0.349665 < 0.582037. Moreover, upper confidence limits of C_{pp} is less than $c_0 = 0.75$ and *p*-value(C_{pp}) = 0.000000 < α = 0.05.

Table 1: The *U.C.L.* for $C_{pp}/\hat{C}_{pp}$, 95%, *m* = 20, based on Range.

n	λ= 0	1	5	10	15	20	25	30
2	1.87299	1.40474	1.09258	1.02163	0.99503	0.98109	0.97251	0.96670
3	1.52051	1.35156	1.15848	1.09814	1.07330	1.05975	1.05122	1.04535
4	1.39897	1.31154	1.18038	1.12994	1.10752	1.09485	1.08670	1.08102
5	1.33542	1.28200	1.18704	1.14464	1.12456	1.11285	1.10517	1.09975
6	1.29560	1.25961	1.18763	1.15164	1.13365	1.12285	1.11565	1.11051
7	1.26803	1.24215	1.18569	1.15481	1.13864	1.12869	1.12194	1.11707
8	1.24760	1.22810	1.18262	1.15586	1.14127	1.13208	1.12576	1.12115
9	1.23176	1.21655	1.17912	1.15573	1.14250	1.13400	1.12808	1.12371
10	1.21908	1.20689	1.17554	1.15492	1.14289	1.13501	1.12944	1.12531

Table 2: The *U.C.L.* for $C_{pp}/\hat{C}_{pp}$, 95%, m = 25, based on Range.

n	λ= 0	1	5	10	15	20	25	30
2	1.73938	1.30454	1.01464	0.94875	0.92405	0.91110	0.90314	0.89774
3	1.44973	1.28865	1.10456	1.04703	1.02334	1.01042	1.00228	0.99669
4	1.34730	1.26310	1.13679	1.08821	1.06662	1.05441	1.04657	1.04110
5	1.29316	1.24144	1.14948	1.10843	1.08898	1.07764	1.07020	1.06496
6	1.25903	1.22406	1.15411	1.11914	1.10165	1.09116	1.08416	1.07917
7	1.23530	1.21009	1.15508	1.12500	1.10925	1.09955	1.09298	1.08824
8	1.21765	1.19863	1.15424	1.12812	1.11388	1.10491	1.09874	1.09424
9	1.20395	1.18909	1.15250	1.12963	1.11671	1.10840	1.10261	1.09834
10	1.19296	1.18103	1.15035	1.13017	1.11840	1.11068	1.10524	1.10119

Table 3: The max. critical values for c_{pp}/c_0, 95%, m = 20, based on Range.

n	λ=0	1	5	10	15	20	25	30
2	0.53391	0.71187	0.91527	0.97883	1.00500	1.01928	1.02826	1.03444
3	0.65768	0.73988	0.86320	0.91063	0.93171	0.94362	0.95128	0.95662
4	0.71481	0.76246	0.84718	0.88500	0.90292	0.91337	0.92022	0.92505
5	0.74883	0.78003	0.84243	0.87364	0.88924	0.89860	0.90484	0.90929
6	0.77184	0.79390	0.84201	0.86832	0.88211	0.89059	0.89634	0.90048
7	0.78863	0.80506	0.84339	0.86594	0.87824	0.88599	0.89131	0.89520
8	0.80154	0.81426	0.84558	0.86515	0.87622	0.88333	0.88829	0.89194
9	0.81185	0.82199	0.84809	0.86526	0.87527	0.88183	0.88646	0.88991
10	0.82029	0.82857	0.85067	0.86586	0.87497	0.88105	0.88539	0.88865

Table 4: The max. critical values for c_{pp}/c_0, 95%, m = 25, based on Range.

n	λ= 0	1	5	10	15	20	25	30
2	0.57492	0.76656	0.98557	1.05402	1.08220	1.09757	1.10725	1.11390
3	0.68978	0.77601	0.90534	0.95508	0.97719	0.98969	0.99772	1.00332
4	0.74222	0.79170	0.87967	0.91894	0.93755	0.94840	0.95551	0.96052
5	0.77330	0.80552	0.86996	0.90218	0.91829	0.92796	0.93440	0.93900
6	0.79426	0.81696	0.86647	0.89355	0.90773	0.91646	0.92237	0.92664
7	0.80952	0.82639	0.86574	0.88889	0.90151	0.90946	0.91493	0.91892
8	0.82125	0.83429	0.86637	0.88643	0.89777	0.90505	0.91013	0.91387
9	0.83060	0.84098	0.86768	0.88524	0.89549	0.90220	0.90694	0.91047
10	0.83825	0.84672	0.86930	0.88482	0.89414	0.90035	0.90478	0.90811

Table 5 : The p-value for C_{pp}, m = 20,λ= 0, based on Range.

$n\backslash W$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
2	0.00000	0.00000	0.00011	0.00177	0.01058	0.03631	0.08804	0.16872	0.27342	0.39170	0.51160
3	0.00000	0.00000	0.00000	0.00001	0.00040	0.00445	0.02403	0.07957	0.18604	0.33752	0.50795
4	0.00000	0.00000	0.00000	0.00000	0.00002	0.00062	0.00728	0.04070	0.13399	0.29988	0.50641
5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00010	0.00245	0.02231	0.10057	0.27161	0.50557
6	0.00000	0.00000	0.00000	0.00000	0.00000	0.00002	0.00088	0.01276	0.07742	0.24883	0.50501
7	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00034	0.00759	0.06087	0.23001	0.50461
8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00014	0.00465	0.04863	0.21404	0.50430
9	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00006	0.00294	0.03950	0.20044	0.50406
10	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00003	0.00190	0.03246	0.18854	0.50387

Table 6 : The p-value for C_{pp}, m = 25,λ= 0, based on Range.

$n\backslash W$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
2	0.00000	0.00000	0.00002	0.00052	0.00478	0.02172	0.06385	0.13966	0.24764	0.37648	0.51033
3	0.00000	0.00000	0.00000	0.00000	0.00009	0.00168	0.01332	0.05703	0.15783	0.31785	0.50708
4	0.00000	0.00000	0.00000	0.00000	0.00000	0.00015	0.00313	0.02554	0.10722	0.27759	0.50573
5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00002	0.00081	0.01223	0.07590	0.24722	0.50497
6	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00023	0.00618	0.05540	0.22314	0.50447
7	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00007	0.00327	0.04146	0.20351	0.50411
8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00002	0.00180	0.03165	0.18706	0.50384
9	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00102	0.02460	0.17314	0.50363
10	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00060	0.01941	0.16111	0.50346

Table 7 : The p-value for C_{pp}, m = 20,λ=1, based on Range.

$n\backslash W$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
2	0.00000	0.00000	0.00001	0.00025	0.00177	0.00721	0.02067	0.04655	0.08804	0.14598	0.21864
3	0.00000	0.00000	0.00000	0.00000	0.00010	0.00130	0.00834	0.03252	0.08878	0.18604	0.31917
4	0.00000	0.00000	0.00000	0.00000	0.00001	0.00023	0.00320	0.02067	0.07811	0.19852	0.37546
5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00005	0.00126	0.01303	0.06620	0.19956	0.40921
6	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00051	0.00820	0.05518	0.19522	0.43074
7	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00021	0.00522	0.04590	0.18872	0.44537
8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00009	0.00337	0.03823	0.18136	0.45578
9	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00004	0.00222	0.03207	0.17402	0.46350
10	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00002	0.00148	0.02704	0.16680	0.46938

Table 8 : The *p*-value for C_{pp}, m = 25,λ=1, based on Range.

$n \backslash W$	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
2	0.00000	0.00000	0.00000	0.00005	0.00052	0.00297	0.01089	0.02943	0.06385	0.11744	0.19015
3	0.00000	0.00000	0.00000	0.00000	0.00002	0.00037	0.00364	0.01926	0.06508	0.15783	0.29786
4	0.00000	0.00000	0.00000	0.00000	0.00000	0.00005	0.00114	0.01121	0.05614	0.17104	0.36004
5	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00036	0.00634	0.04585	0.17196	0.39744
6	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00012	0.00359	0.03679	0.16758	0.42149
7	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00004	0.00207	0.02945	0.16107	0.43790
8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00121	0.02364	0.15377	0.44961
9	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00001	0.00073	0.01910	0.14644	0.45829
10	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00044	0.01554	0.13933	0.46490

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