使用另一能力組合指標評估六標準差管理

Using an Alternative Match Index to Assessing Six-sigma Management

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Abstract

Six-Sigma approach is a business philosophy of driving behaviour by making an organization's values explicit in its compensation system and a business strategy of cutting costs and boosting customer satisfaction. It is vital to investigate each part of the business in such a way everyone can understand the causes of variation that can be led to improvement in both process and performance. In this paper, focuses on the elucidation of the "Six-Sigma process quality", by another incapability index C_{pp}, to decide whether a given process meets the capability requirement under quality control. Further, to rapidly facilitate in more general situations suitable for engineers to apply, we also provide a comprehensive set of across multiple process indices matches criteria analysis and by rendering enhanced decision-making ability, corresponding the indices values of C_p, C_{pp} and *ppm* to a $\pm K\sigma$, with $\mu = T$, $\mu \neq T$, and to a shifted process $\pm K\sigma$, with the indices values of match (C_p, C_{pp}).

***Keywords*：**Incapability indices; Sigma quality level; Six-sigma; Parts per million

1. Introduction

In 1988, Motorola developed and vigorously pursued a quality management program called Six-Sigma, and it attributes much of its quality improvement to his program. Caulcutt(2001) pointed that Minitab Inc defines the Six-Sigma is as:

「Six-Sigma is an information-driven methodology for reducing waste, increasing customer satisfaction and improving process, with a focus on financially measurable results.」An alternative definition, which was used in Motorola, offers a rather different perspective:

「Six-Sigma is a business philosophy of driving behavior by making an organization's values explicit in its compensation system and a business strategy of cutting costs and boosting customer satisfaction.」

Six-Sigma can be defined in several ways. Clearly, none of the definitions is complete. Perhaps it is not possible to define Six-Sigma in one simple sentence. Basically, Blakeslee(1999) point out that it's a high-performance, customer-driven approach to analyzing the root causes of business problems and solving them, and provides an overall framework for quality management. The major fundament components of a typical Six-Sigma program are improvement process, quality measurement, quality initiatives and improvement tools. The goal of a Six-Sigma program is to improve customer satisfaction through reducing and eliminating defects. In

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1990, Peter J. Billington and Ahmad Ahmadian were denoted that it uses several statistical measures to characterize defect levels and process capabilities, and then relies on continuously improve processes throughout the organization, reducing sources of variation and improving quality and productivity. Broadly speaking, it's a way of measuring processes; a goal of near-perfection, represented by 3.4 parts per million (*ppm*) or Defects per Million Opportunities (DPMO); an approach to changing the culture of an organization.

Six-Sigma relies on normal distribution theory to predict defect rates. Assume the measurement follows a normal distribution and that the mean (μ) and standard deviation (σ) are known or have been estimated from sample statistics under statistical control. Moreover, McFadden (1993) was point out some key assumptions in Six-Sigma quality measurement that warrant examination to be included is:

(1). Each process parameter can be characterized by a normal distribution.

(2). A second assumption is that a shift in the process mean of 1.5σ from nominal is likely to occur, and the design goal of Six-Sigma processes (C_p = 2, $C_{pk} \geqq 1.5$) is necessary to provide a safe margin against such shifts.

(3). Another assumption is that defects are randomly distributed throughout units, and parts and process steps are independent of each other.

(4). A final assumption is that the process mean and standard deviation are known, and C_p and C_{pk} are parameters with point values. In facts, μ and σ must normally be estimated from sample statistics.

In Table 1, we show the indices C_p, C_{pk}, *ppm* defect rates vs. sigma level (or sigma quality level, ±*K*σ) for processes with the mean centered at the nominal (or target, *T*) value, or shift ±1.5σ. The ± sign is necessary as the specification width stretches from –*K*σ to +*K*σ.

Table 1 Corresponding the indices values of C_p, C_{pk} and *ppm* to a ±*Kσ*, with μ=*T* and μ≠*T*

Sigma level	Centered Process		Shifted (±1.5σ) Process	
Kσ	Design Capability C_p	*ppm*	Manufacturing C_{pk}	*ppm*
2*σ*	0.6667	45500	0.1667	308770
3*σ*	1	2700	0.5	66811
4*σ*	1.3333	63.3721	0.8333	6210
4.5*σ*	1.5	6.8016	1	1350
4.645σ	1.5483	3.4	1.0483	831
5*σ*	1.6667	0.5733	1.1667	233
6*σ*	2	0.0020.	1.5	3.4
7*σ*	2.3333	0.0026 ppb	1.8333	0.0190

Note：1ppm = 10^{-6}，1ppb = 10^{-9}

The Table 1 informs us that the Six-Sigma process will give us 0.002 ppm, if it is on target, and 3.4 ppm if it is 1.5*σ* off target when the measurement controlled process follows a normal distributed. Translating this performance standard into the language of capability indices, a Six-Sigma process has a C_p = 2, and a $C_{pk} \geqq 1.5$ provided any shifts in mean do not exceed 1.5*σ*. These 3.4 ppm defects are sometimes called "virtually zero defects".

Perhaps it would be easier to define Six-Sigma by describing the characteristics that are shared by the companies in which it has succeeded. One of these common characteristics is a widespread focus on process and the existence of a company-wide language for describing the capability of processes. Moreover, process capability indices are used to measure on whether a manufacturing process meet a set of requirement preset in the workshop, which is designed to quantify the relation between the actual performance of the process and its specified requirement.

The construction of this paper is as follows. In section 2, we brief demonstrated of Six-Sigma program, derived a measure off targetness and process yield from a generalization class of capability indices $C_p(u, v)$, $u = 1$, $v = 0$, with $m = T$. Section 3, which tie in two indices C_p and C_{pp} together, derive a rule to satisfy Six-Sigma program. The final sections, we also provide a comprehensive set of the other indices matches rule, (C_p, C_{pk}), or (C_p, C_{pp}), through multi-choice and by rendering enhanced decision-making ability for user to apply.

2. The Six-Sigma metrics for C_p and $C_p(1, 0)$

In 1995, Vännman proposed a more cautious approach, which utilizes two real-valued parameters u and v, and to be considered the index

$$C_p(u, v) = \frac{d - u|\mu - m|}{3\sqrt{\sigma^2 + v(\mu - T)^2}}. \tag{2.1}$$

Vännman(1995) considers the case $T = m$, which is quite common in practical application when we have two-sided specification limits. That is, (2.1) is just the same of following :

$$C_p(u, v) = \frac{d - u|\mu - T|}{3\sqrt{\sigma^2 + v(\mu - T)^2}}, \tag{2.2}$$

where [USL, LSL] is the specification interval, $d=(USL–LSL)/2$ is the half-length of the specification interval, $m=(USL+LSL)/2$ is the center of the specification interval, T is the target value, μ is the process mean, and σ is the process standard deviation and under statistical controlled conditions.

Assume that the measurements are the observed randomly from $N(\mu, \sigma^2)$ under statistical control condition. Then, the capability index $C_p(u, v)$ and C_p are directly tied together by the relation

$$C_p(u, v) = \frac{d - u|\mu - T|}{3\sqrt{\sigma^2 + v(\mu - T)^2}} = C_p \times \frac{1 - \dfrac{u|\mu - T|}{d}}{\sqrt{1 + v\left(\dfrac{\mu - T}{\sigma}\right)^2}}, \tag{2.3}$$

where $C_p = C_p(0,0) = d / (3\sigma)$.

Notably, solving ε for (2.3), yields,

$$\varepsilon = |\mu - T| = d\left(\frac{u}{u^2 - 9vC_p^2(u,v)} \pm \sqrt{\frac{\left(\dfrac{C_p(u,v)}{C_p}\right)^2 - 1 + \dfrac{u^2}{u^2 - 9vC_p^2(u,v)}}{u^2 - 9vC_p^2(u,v)}}\right)$$

$$= 3\sigma C_p \left(\frac{u}{u^2 - 9vC_p^2(u,v)} \pm \sqrt{\frac{\left(\frac{C_p(u,v)}{C_p}\right)^2 - 1 + \frac{u^2}{u^2 - 9vC_p^2(u,v)}}{u^2 - 9vC_p^2(u,v)}} \right), \qquad (2.4)$$

where ε is denotes the process mean deviates from the target and measures of "off targetness".

When $C_p(u, v) = c$ and with $m = T$, the process mean is given by

$$\mu = T \pm 3\sigma C_p\, g(c, C_p, u, v), \qquad (2.5)$$

where $g(c, C_p, u, v) = \left(\frac{u}{u^2 - 9vc^2} \pm \sqrt{\frac{\left(\frac{c}{C_p}\right)^2 - 1 + \frac{u^2}{u^2 - 9vc^2}}{u^2 - 9vc^2}} \right)$.

For the bilateral specifications limits situation and have symmetric tolerance (*i.e.*, $(LSL, T, USL) = (-K\sigma, 0, K\sigma)$), then the process yield is given by

$$\%Yield = P(LSL < X < USL) = \Phi\left(\frac{USL-\mu}{\sigma}\right) - \Phi\left(\frac{LSL-\mu}{\sigma}\right)$$

$$= \Phi(K - 3C_p\, g(c, C_p, u, v)) + \Phi(K + 3C_p\, g(c, C_p, u, v)) - 1, \qquad (2.6)$$

where Φ denotes the standard normal cumulative distribution function.

For $(u, v) = (1, 0)$, the capability index $C_p(1, 0) = C_{pk}$ is given by

$$C_{pk} = (d - |\mu - T|)/(3\sigma) = (1 - |\mu - T|/d)C_p. \qquad (2.7)$$

For the quantity of C_p, only the spread of the process; C_{pk} is used concurrently to consider the spread and mean shift of the process. From (2.4) or (2.7), and $C_{pk} = c$, solving ε for (2.7), yields,

$$\varepsilon = |\mu - T| = d\,(1 - c/C_p). \qquad (2.8)$$

For $\varepsilon = 0$, the process mean is centered on target, $\mu = T$, $C_{pk} = C_p$, and the sigma quality level is $\pm K\sigma$. When $\varepsilon = d$, the process mean is located at one of the specification limits and $C_{pk} = 0$, this means that the process is not adequate with respect to the production tolerances and needs to be modify step by step until the required level of the consumer's quality is met. Therefore, when $0 < \varepsilon < d$, the process mean is located between the target and a specification limit.

Consequently, so that process yield (2.6) can be replayed by,

$$\%Yield = P(LSL < X < USL) = \Phi\left(\frac{USL-\mu}{\sigma}\right) - \Phi\left(\frac{LSL-\mu}{\sigma}\right)$$

$$= \Phi(K - 3(C_p - C_{pk})) + \Phi(K + 3(C_p - C_{pk})) - 1. \qquad (2.9)$$

Obviously, a Six-Sigma quality level is said to translate to $C_p = 2$ (*i.e.*, $d = 6\sigma$) and $C_{pk} = 1.5$, then the process mean deviates from the target $\varepsilon = 1.5\sigma$. That is, to achieve this basic goal of a Six-Sigma program might then be to produce no more than 3.4 defects per million parts or process steps if the process mean were to shift by as much as 1.5σ, $\mu = T \pm 1.5\sigma$.

When we considering symmetric tolerance, $(LSL, T, USL) = (-6\sigma, 0, 6\sigma)$, the process yield is given by

$\%Yield = P(-6\sigma < X < 6\sigma) = \Phi(4.5) + \Phi(7.5) - 1 = 0.9999966.$

We notice, from Table 1, this percentage non-conforming 3.4 ppm, which is a half of the probability non-conforming in C_p = C_{pk} = 1.5 with a centered process. And, according to Table 1, for deducing the process quality from the defect rate, the process shift is decisive 3.4 ppm defects correspond both to a ±6σ process with a shift of ±1.5 and to an unshifted ±4.645σ process. It can be seen in Table 1 that, by improving a ±3σ process (with ±1.5σ shift) to a ±4σ process (with ±1.5σ shift), the defects are reduced from 66811 ppm to 6210 ppm, it means that this process need to be about 10 times never-ending improvement effort. Similarly, from ±4σ to ±5σ process, it needs to be about 30 times improvement effort; and from ±5σ to ±6σ process, it needs to be about 70 times continuous improvement effort.

The comparison the defect rate of a centered process and of a process shifted by ±1.5σ is depicted graphically in Figure 1.

Moreover, in Table 2, it can be seen that a process quality has a C_p = 2, and a $C_{pk} \geqq 1.5$ provided any shifts in mean do not exceed 1.5σ exhibits ≦3.4 ppm defects. Similarly, as C_p = 2.3, then $C_{pk} \geqq 1.8$ provided any shifts in mean do not exceed 1.5σ expresses ≦3.4 ppm defects. For any centered process, C_p = C_{pk}; for processes with process shifts, $C_p > C_{pk}$ holds. The further C_{pk} is to C_p, the larger is the process shift.

As mentioned before, a Six-Sigma program corresponds to a defect rate of, at most, 3.4 ppm, only if a maximally allowed process shift of ±1.5σ is taken into account; that is C_p = 2, $C_{pk} \geqq 1.5$ and $\mu \leqq T \pm 1.5\sigma$. Besides, at the same Six-Sigma quality requirement, and as the C_p value is 2.3, then a $C_{pk} \geqq 1.8$, if a maximally allowed process shift of $\mu \leqq T \pm 1.5\sigma$.

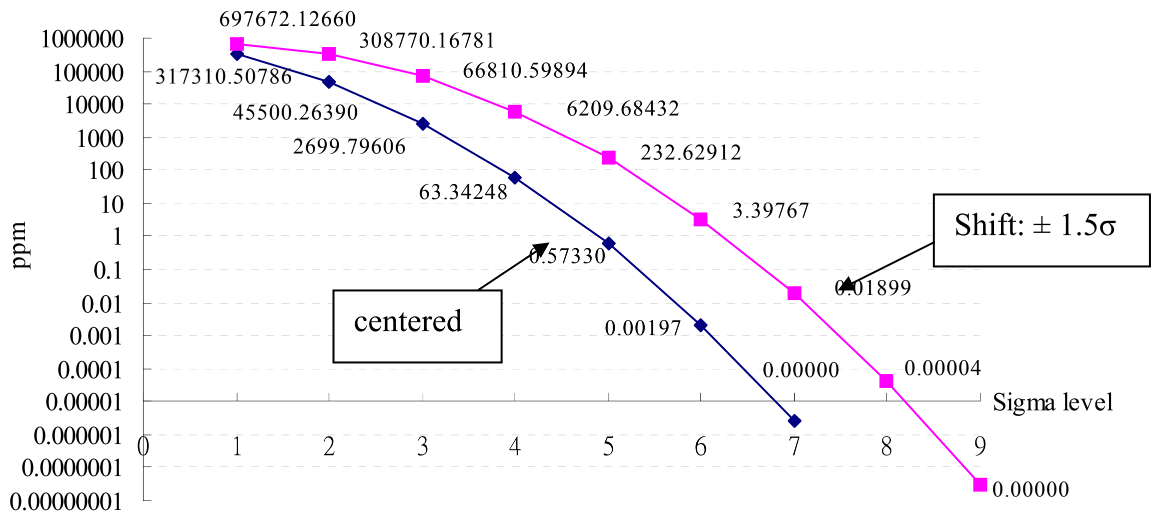

Figure 1 Sigma level vs. ppm

Table 2 The correspond to a shifted process ± *K*σ, with the indices values of match (C_p, C_{pk})

		C_p value											
K value		1	1.2	1.33	1.4	1.5	1.67	1.8	2	2.1	2.2	2.3	2.33
C_{pk} value	0.67	1	1.6	2	2.2	2.5	3	3.4	4	4.3	4.6	4.9	4.99
	1	0	0.6	1	1.2	1.5	2	2.4	3	3.3	3.6	3.9	3.99
	1.2		0	0.4	0.6	0.9	1.4	1.8	2.4	2.7	3	3.3	3.39
	1.33			0	0.2	0.5	1	1.4	2	2.3	2.6	2.9	2.99
	1.4				0	0.3	0.8	1.2	1.8	2.1	2.4	2.7	2.79
	1.5					0	0.5	0.9	1.5	1.8	2.1	2.4	2.49

	1.6		0.2	0.6	1.2	1.5	1.8	2.1	2.19
	1.67		0	0.4	1	1.3	1.6	1.9	1.99
	1.8			0	0.6	0.9	1.2	1.5	1.59
	1.83				0.51	0.81	1.11	1.41	1.5
	1.9				0.3	0.6	0.9	1.2	1.29
	2				0	0.3	0.6	0.9	0.99
	2.1					0	0.3	0.6	0.69
	2.2						0	0.3	0.39
	2.3							0	0.09

3. Tying C_p and C_{pp} together metrics

In 1995, Greenwich and Jahr-Schaffrath introduced another incapability index C_{pp}, which is the simple transformation of the index C^*_{pm} proposed by Chan et al.(1988), and defined as follows:

$$C_{pp} = (1/C^*_{pm})^2 = \frac{(\mu - T)^2}{D^2} + \frac{\sigma^2}{D^2} = C_{ia} + C_{ip} , \tag{3.1}$$

where $D = min\{ USL - T, T - LSL \}$. Let $(\mu - T)^2/ D^2 = C_{ia}$ (inaccuracy index) and $\sigma^2/ D^2 = C_{ip}$(imprecision index).

The incapability index C_{pp} provides an uncontaminated separation between the information from the accuracy and the precision of the process, which is not available for index C^*_{pm}. The smaller value and the larger value of C_{pp} imply a more capable process and a less capable process respectively. The non-zero value of C_{pp} indicates the degree of incapability of the process. These sub-indices (C_{ia} and C_{ip}) also provide the proportions of the process incapability due to the departure of the process mean from the target and the process variation, respectively.

$$C_{pp} = \frac{(\mu - T)^2/\sigma^2 + 1}{C_p^2} . \tag{3.2}$$

From (2.1) or (3.1), and $C_{pp} = c$, solving ε for (3.1), yields,

$$\varepsilon = |\mu - T| = \sigma\sqrt{C_p^2 \times c - 1} . \tag{3.3}$$

Consequently, so that process yield (2.9) can be replayed by,

$$\%\text{Yield} = P(LSL < X < USL) = \Phi\left(\frac{USL-\mu}{\sigma}\right) - \Phi\left(\frac{LSL-\mu}{\sigma}\right)$$

$$= \Phi\left(K - \sqrt{C_p^2 \times c - 1} \right) + \Phi\left(K + \sqrt{C_p^2 \times c - 1} \right) - 1. \tag{3.4}$$

We show these results value in Table 3. It informs us that the Six-Sigma process will give us 3.4 ppm if it is 1.5σ off target when a Six-Sigma measurement process has a $C_p = 2$, and a $C_{pp} \leqq 0.8125$ provided any shifts in mean do not exceed 1.5σ.

Table 3 Corresponding the indices values of C_p, C_{pp} and *ppm* to a $\pm K\sigma$, with $\mu = T$ and $\mu \neq T$

Sigma level	Centered Process			Shifted (±1.5σ) Process	
	Design Capability			Manufacturing	
$K\sigma$	C_p	C_{pp}	*ppm*	C_{pp}	*ppm*
2σ	0.6667	2.25	45500	7.3125	308770

3σ	1	1	2700	3.25	66811
4σ	1.3333	0.5625	63	1.8281	6210
4.5σ	1.5	0.4444	6.8	1.4444	1350
4.645σ	1.5483	0.4171	3.4	1.3557	831
5σ	1.6667	0.36	0.5733	1.17	233
6σ	2	0.25	0.002	0.8125	3.4
7σ	2.3	0.1837	0.0026 ppb	0.5969	0.0190

And then, in Table 4, it can be seen that a process quality has a C_p = 2, and a C_{pp}≦ 0.8125 provided any shifts in mean do not exceed 1.5σ exhibits≦3.4 *ppm* defects. For any centered process, C_p≧C_{pp}, sigma level≧3σ; for processes with process shifts, C_p>C_{pp} sigma level≧4.5σ. The further C_{pp} is to C_p, the smaller is the process shift.

Table 4 The correspond to a shifted process ± *K*σ, with the indices values of match (C_p, C_{pp})

K value		C_p value									
		1	1.2	1.4	1.5	1.6	1.8	2	2.1	2.2	2.3
	1	0	0.6633	0.9798	1.1180	1.2490	1.4967	1.7321	1.8466	1.9596	2.0712
	0.9		0.5441	0.8741	1.0124	1.1419	1.3842	1.6125	1.7231	1.8319	1.9393
C_{pp}	0.8125		0.4123	0.7697	0.9100	1.0392	1.2777	1.5	1.6072	1.7125	1.8161
value	0.8		0.3899	0.7537	0.8944	1.0237	1.2617	1.4832	1.5900	1.6947	1.7978
	0.7		0.0894	0.6099	0.7583	0.8899	1.1261	1.3416	1.4446	1.5453	1.6441
	0.6			0.4195	0.5916	0.7321	0.9716	1.1832	1.2830	1.3799	1.4744
	0.5				0.3536	0.5292	0.7874	1.0000	1.0977	1.1916	1.2826
	0.4					0.1549	0.5441	0.7746	0.8741	0.9675	1.0564
	0.3							0.4472	0.5683	0.6723	0.7662
	0.2										0.2408
	0.1										

As precious statement, a Six-Sigma program corresponds to a defect rate of, at most, 3.4 ppm, only if a maximally allowed process shift of ±1.5σ is taken into account; that is C_p = 2, C_{pp}≦0.8125 and μ≦T± 1.5σ. In addition, as C_p = 2.1, and a defect rate of, at most, 3.4 ppm, then a C_{pp}≦ 0.7 for μ≦T± 1.5σ.

4. Conclusion

The philosophy of Six-Sigma recognizes that there is direct correlation between the number of product defects, wasted operating cost, and the level of customer satisfactions. From previous mentioned, it becomes obvious that a metric describes how well a process meets requirements is process capability, And a Six-Sigma quality level is said to translate to process capability index value for C_p and C_{pk} requirement of 2 and 1.5, respectively. To increase quality and yield, process variations must be eliminated and prevailing ± 4σ processes must be improved substantially to achieve this basic goal of a Six-Sigma program might then be to product no more than 3.4 defects process steps if the process mean were to shift by as much as 1.5σ. Further, we also provide a comprehensive set of the other indices matches rule, C_p = 2 and C_{pk}≧1.5, or C_{pp}≦ 0.8125, through multi-choice and by rendering enhanced decision-making ability for practicers to apply.

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