

# Regime switching cointegration tests for the Stock Indices and corresponding Futures Prices: Evidence from MSCI Taiwan and Hang-Seng Equity Indices

Jo-Yu Wang\*, Chia-Yen Wei

## Abstract

In this paper, two non-linear cointegration models, including a modified cointegration method considering regime shifts and a threshold cointegration model, are compared with each other. Also, the results of the two non-linear models are compared with the original simple cointegration model to check if they are better. Thus, three models are applied to estimate the long-term relationship between two equity index returns and their corresponding futures index returns. Two data sets including MSCI Taiwan equity index and Hang-Seng equity index, from the first of 2003 to the end of 2010, are employed in this study. In the empirical test, we obtain two major results. The first, the evidence shows that the cointegration exists between the index returns and corresponding index futures returns. Generally, the estimation performance of non-linear cointegration model is better than the one of linear cointegration. The second, the cointegration model with regime shifts generally performs better than threshold cointegration model, although the latter one really has some advantage in examining the asymmetric effect of residual sequence. Furthermore, the regime switching model captures the impact of subordinated-debt crisis in 2007 to 2008.

**Keywords:** Cointegration, Index Futures, Regime Switching.

---

Jo-Yu Wang (corresponding), Lecturer, Department of applied financial management, Hsiuping University of Science and Technology.

Chia-Yen Wei, Lecturer, Department of Electronic Engineering, Hsiuping University of Science and Technology.

Received 18 March 2012; accepted 29 July 2013



# 區域轉換下之共整合檢定：以 MSCI 台灣股價指數與恆生指數為例

王若愚、魏嘉延

## 摘要

本篇文章比較兩種非線性之共整合模型(包括考慮狀態變數改變的修正性共整合模型及不對稱門檻共整合模型)對於股票指數與股票指數期貨之間的長期關係衡量,同時也比較此二種模型是否比原始簡單共整合模型更具有較好的衡量能力。並利用三種不同共整合模型來衡量 2003 年至 2010 年之間台灣 MSCI 指數與恆生股價指數及此二指數之期貨指數的長期關係。本篇文章主要的實證結論有兩項。第一、實證中說明股價指數與期貨指數之間具有共整合之效果,亦即兩者之間具有長期的關係,且整體而言,非線性共整合模型對於衡量股價指數與期貨指數之間的關係效果較好(與線性共整合模型相比)。第二、在兩種非線性共整合模型中,於衡量股價指數與期貨指數之間的長期關係而言,考慮區域轉換共整合模型之衡量績效比門檻共整合模型更好。同時,區域轉換共整合模型也偵測到 2007 到 2008 年次順位債券風暴對於此而金融市場所產生的衝擊與影響。

**關鍵詞：**共整合、指數期貨、區域轉換。



## 1. INTRODUCTION

Understanding the relationship between spot and futures index prices helps us uncover the price formation of a spot index in relation to its derivative. The studies of the relationships between spot and futures prices have focused mostly on two concepts. The first is based on the cost-of-carry assumption that the difference between concurrent spot and futures prices is due to relevant costs, such as foregone interest expenses, warehousing costs, and convenience yield, etc. for holding the spot goods. The other is to view the futures price as the composition of expected risk premium and forecasted future spot price. Thus, even though the futures price maintains its equivalence to the sum of spot price and storage costs it should still contain risk rewards to risk takers. The two views of point on futures prices do not compete with each other but rather work together to explain the formation and deviations of the futures price from the observing spot price.

Based on the cost-of-carry theory, empirical studies have indicated that mispricing behaviors soon disappear in a mature futures market. In addition, even though there are discrepancies between theoretical and actual futures prices, the constitutional costs may be able to

eliminate possible arbitrage opportunities.<sup>1</sup> This implies that spot and futures prices possess a long-run relationship. Ever since the seminal paper of Engle and Granger(1987) was published, which used cointegration methods to estimate long-term relationships among economic variables, this model has received much attention. The primary merit of using cointegration methods is to retain the information content of variables, especially non-stationary variables, without differencing non-stationary variables. The major assumptions behind the usual cointegration methods rely on the stability of the long-term relationships. Some researchers have used cointegration methods to investigate the long-run relationship between spot and futures prices. In fact, most studies (see Lai and Lai, 1991, Chowdhury, 1991, Wahab and Lashgari, 1993, Kroner and Sultan, 1993, Antoniou and Holmes, 1996, Ghosh and Clayton, 1996, Chen, Finney, and Lai, 2005) have suggested that a cointegration relationship exists between spot and futures prices.

However, the structural changes might occur in the long-term relationships between the spot and futures. Also, Kasuya

---

<sup>1</sup> See Figlewski(1984), Bhatt and Cakici(1990), Saunders and Mahajan(1998), Lim(1992), and Chan and Chung(1993), etc.



(2005) indicated that a traditional linear model cannot adequately explain real economic conditions that vary rapidly time wise, implying the gap in the literature of this area. Kuo and Lu (2005) argued that a traditional linear model with fixed parameters seems to disregard information from another regime when the structural changes during the estimation period exist. Based on that concept, a cointegration method that considers such changes may prove useful. To this, Balke and Fomby (1997) firstly introduced threshold cointegration to integrate non-linearity into cointegration. Specifically, their model allows a mechanism of nonlinear adjustment to capture the changes in the long-term relationship according to the market conditions. The model has attracted significant attention and a large number of articles were generated based on similar concept, including Baum et al. (2001), Enders and Siklos (2001), Martens et al. (1998), Obstfeld and Taylor (1997), Taylor (2001), and Chen, Finney, and Lai (2005). Enders and Siklos (2001) extended this line to a powerful threshold cointegration model with asymmetric error correction.

In the empirical study of long-term relationship between financial markets, Willis (2003) concluded that structural change in the economy affects the

price-setting behavior of investors. In other words, the risk premium is adjusted when investors change their beliefs due to the economic shift occurs. Alternatively, some might use other method to perform the nonlinearity. Investigating the S & P 500, the MMI and the Toronto 35 index futures, Park and Switzer (1995) suggested that a cointegration model with stochastic variances performs better than a model with constant variance specification and time-varying hedge ratios as a trading strategy. Therefore, the long-run relationship between spot and futures prices may be changed due to the condition of economy. Generally, the shifts in regimes of the economy could be derived from changes in investors' beliefs, adjustments in the macroeconomic environment, innovations in technology, or deviations in the worldwide political situation. For example, the market crash of the NYSE in October 1987 brought the linkage among international financial markets much closer than before. Hamilton (1989) suggested a Markov-switching model implying that the market conditions might change frequently based on the likelihood of each time horizon. Filardo (1994) further extended the model to a time-varying Markov-switching (TVMS) model. Chen and Lin (2000) also applied time-varying Markov-switching model to



evaluate the usefulness of the coincident and leading indices in dating the business cycle and in predicting future Taiwan's GDP, and they suggested that the method of two-state TVMS is appropriate for Taiwanese GDP.

Stimulated by Chen and Lin (2000), a cointegration system with regime shifts is used to describe the dynamic relationship between equity index and futures index in this paper. Besides, the results of the model above are compared with the original cointegration model proposed by Engle and Granger (1987) and Enders and Siklos' (2001) threshold cointegration model which is another type of nonlinear cointegration model. The main contribution of this study is to provide a clear comparison between simple cointegration model and non-linear cointegration models. Besides, by using MSCI Taiwan and Hang-Seng equity indices from 02 January 2003 to the end of 2010 in the empirical test, the evidence generally shows that there is a long-run relationship between index return and futures return. Both of the two non-linear cointegration models (i.e., regime shifts and threshold model) provide better performance than the original cointegration model based on the criteria, mean square error. And the former is the best one in this study. The outcome of regime shifts model indicates that the

MSCI Taiwan index tends to stay in the lower premium state more often, but Hang-Seng Index generally stays in the high premium state. The break point of the MSCI Taiwan is around the second or third quarter in 2006, which might be associated with the subordinated-debt crisis in 2007-2008.

The structure of this paper is organized as follows: Section 2 presents the cointegration system in regime switching and the estimation specification. Section 3 describes the data used in this paper. Section 4 discusses the empirical results. Section 5 is the conclusion.

## 2. COINTEGRATION IN REGIME SWITCHING

### 2.1 The Regime Switching Model

Let  $S_t$  be the spot index and  $F_t$  be the index futures price at time  $t$ . It is assumed that the spot index follows a Gaussian random walk with a drift in which the drift and innovation variance depend upon the state of the economy.

$$S_{t+1} = S_t + u_0(1 - Z_{t+1}) + u_1 Z_{t+1} + [\sigma_0(1 - Z_{t+1}) + \sigma_1 Z_{t+1}] \eta_{t+1} \quad (1)$$

Where  $s_t$  and  $s_{t+1}$  are the logarithms of  $S_t$  and  $S_{t+1}$ ,  $Z_{t+1}$  is an indicator variable equal to 1 when the economy is in state 2 and 0 otherwise, and  $\{\eta_{t+1}\}$  is distributed as a sequence of



i.i.d. $N(0,1)$ . The regime  $z_t$  is modeled as the first-order Markov chain in a two-state<sup>2</sup> economy as follows :

$$Pr(z_t = 0 | z_{t-1} = 0) = p_{11} = p$$

$$Pr(z_t = 1 | z_{t-1} = 0) = p_{21} = 1 - p$$

$$Pr(z_t = 1 | z_{t-1} = 1) = p_{22} = q$$

$$Pr(z_t = 0 | z_{t-1} = 1) = p_{12} = 1 - q$$

where  $Pr(\cdot)$  denotes the conditional event probability, and  $p_{ij}$  is the transition probability from state  $j$  to state  $i$ .

In addition, in a rational market we can present the relationship between spot and futures prices as

$$E_t(S_{t+1}) = F_t + \lambda \quad (2)$$

Where  $\lambda$  represents the risk premium of a futures price as indicated in Fama and

French (1987). With equations (1) and (2), the expectation of the next period's spot index at time  $t$ ,  $E_t(S_{t+1})$  can be expressed as<sup>3</sup> :

$$S_t = \alpha_i + \beta_i F_t, \quad i = 1 \text{ or } 2 \quad (3)$$

where  $\alpha_i = \frac{\lambda}{R_i}$ ,  $\beta_i = \frac{1}{R_i}$ ,  $R_i = \exp\left(u_i + \frac{1}{2}\sigma_i^2\right)$  and  $i = 1$  is for  $z_{t+1} = 0$  while  $i = 2$  stands for  $z_{t+1} = 1$ . For equation (3), the cointegration parameter changes as regime shifts.

## 2.2 The Estimation Specification

According to the model proposed above, we are able to set up the following estimation specification for the long-run relationship between spot and futures prices:

$$S_t = \alpha_i + \beta_i F_t + u_i, \quad i = 1 \text{ or } 2 \quad (4)$$

where  $u_i$  is a normal distribution with  $E(u_i) = 0$  and  $\text{var}(u_i) = \sigma_{u_i}^2$ . The set of parameter in equation (4) is denoted as  $\Omega = \{\alpha_1, \alpha_2, \beta_1, \beta_2, p_{11}, p_{22}, \sigma_{u1}, \sigma_{u2}\}$ . Hence, the log-likelihood function can be constructed as follows:

$$L(\Omega) = \sum_{t=1}^N \log f(S_t | F_t, \Omega) \quad (5)$$

where  $f(S_t | F_t, \Omega)$  is the conditional probability function for each data point.<sup>4</sup> The estimated parameters can be obtained

<sup>2</sup> How to decide the number of state in Markov switching model has been discussed by several scholars such as Hamilton (1990), Psaradakis and Spagnolo (2003), Cheung and Erlandsson (2005), and Smith, Naik, and Tsai (2006). However, there is no conclusive method to solve this issue. Two and three states Markov switching models have been tried based on the maximum likelihood estimation (MLE) in this paper. The convergence ratio of two states model is around 97.01%, however, before the case of three states is about 69.73%. The phenomenon of low convergence ratio may be attributed to overparameterization of three-state Markov switching model. Besides, the main aim of this study is not to the optimal number of state in Markov switching model but to confirm that non-linear cointegration model is better than the original model by using long-range dataset of MSCI Taiwan index and Hang Seng index. Thus, cointegration model with a two-state Markov switching process is used in this paper.

<sup>3</sup> The detail of the model derivation can be found in Appendix A.

<sup>4</sup> The unconditional densities are set as the initial state probabilities.



by maximizing equation (5).

To test the constancy of the parameters and cointegration with regime shifts, we employed Gregory and Hansen's (1996) tests (hereafter, GH tests). The GH tests consider the null hypothesis of no cointegration against the alternative hypothesis of cointegration with regime shifts. Before constructing GH test statistics, the following equation was run:

$$y_t = \pi_1 + \pi_2 \varphi_{t\tau} + \gamma_1 x_t + \gamma_2 x_t \varphi_{t\tau} + e_t \quad (6)$$

$$\varphi_{t\tau} = \begin{cases} 1, & \text{if } t > [N \times \tau] \\ 0, & \text{otherwise} \end{cases}$$

where  $t = 1, \dots, N$ . In eq. (6),  $\pi_1$  and  $\gamma_1$  are cointegration parameters before the regime switching, and  $\pi_2$  and  $\gamma_2$  are for the changes in the cointegration parameters once there are regime shifts.  $\varphi_{t\tau}$  is a dummy variable that denotes the timing of the regime shift,  $\tau$ . To calculate the necessary statistics from equation (6), the starting point for the structural break needs to be set first. The break point,  $\tau$ , is set up to range from 5% to 85% of observations and is increased by 1% for each iteration. Then, for each  $\tau$ , the equation (6) is computed, and the residuals are reserved for each computation. The reserved residuals obtained from iterations of running equation (6) are utilized to construct the following three test statistics:

$$z_\alpha^* = \inf_{\tau \in T} z_\alpha(\tau)$$

$$z_t^* = \inf_{\tau \in T} z_t(\tau)$$

$$ADF^* = \inf_{\tau \in T} ADF(\tau)$$

Where  $T$  is a compact subset of  $(0,1)$ , and  $z_\alpha$  and  $z_t$  are described in Phillips (1987). Consequently, we are able to reject the null hypothesis whenever the test statistics exceed the critical values.

### 3. DATA

Two sets of data, including MSCI Taiwan Index and Hang-Seng equity index, are applied in this study. Both of MSCI Taiwan index prices and MSCI Taiwan index Futures prices traded in Singapore Exchange (SGX) are collected from *DataStream* including daily spot and nearby<sup>5</sup> futures prices. The spot and futures prices of Hang-Seng equity index are collected from Taiwan Economic Journal (TEJ). For the purpose of avoiding the thin-trading issue, the sample period in this paper is from 2<sup>nd</sup> January 2003 to 31<sup>th</sup> December 2010, totally 1,831 and 1,775 observations for MSCI Taiwan index and Hang-Seng index, respectively. The unit root tests shown in Table 1 confirm that all the original series have unit roots and the

<sup>5</sup> As our best knowledge, there is no standard method of the term "nearby". Kawaller, Koch, and Koch's (1987) set different trading periods prior to expiration as nearby futures prices. Based on their method, we follow the *DataStream*'s definition of nearby futures prices that the futures prices roll to next month contract prices at five trading days (i.e. one week) prior to expiration.



first-order differenced series are stationary. Thus, the data under investigation are integrated with order one. In addition, Table 2 shows the constancy tests of cointegration coefficients between equity index and futures prices for the market data using the cointegration tests of GH. It is obvious that the cointegration relationships for MSCI Taiwan and Hang-Seng equity index are significant instability in GH tests, which allows for a one-time shift. In other words, there is a long-term relationship between the two equity indices and their index futures according to the market condition, implying investors could use index return to forecast the movement of index futures in some circumstances. Also, the estimated breakpoint is 0.26 for the MSCI Taiwan index and 0.34 for Hang-Seng index which is around the second or third quarter in 2006. We may attribute these breaks as a signal anticipating the subordinated-debt crisis happened in the U.S. starting from 2007. We can find that the cointegration relationship with regime shift exists between equity and futures index returns. Consequently, the equity index was cointegrated with its futures index. The empirical evidence also shows that there exist possible regime shifts for the index.

[Insert Table 1 about here]

[Insert Table 2 about here]

## 4. EMPIRICAL RESULTS

The first step of threshold cointegration test is to estimate the parameter in Eq. (4) without any state by using ordinary least squares (OLS). Then, the residuals are used to examine its stationarity, which has been presented in Table 2. Table 3 presents the results of threshold cointegration test proposed by Enders and Siklos (2001) which, in fact, is a threshold autoregressive process (shown below Table 3) in the residual sequence with an unknown threshold value,  $\tau$ . Chan's (1993) method is applied to estimate the threshold value by minimizing the sum of residual sequence. The estimated parameters ( $\hat{\rho}_1$  and  $\hat{\rho}_2$ ) in Table 3 elaborate that equity index return and index futures return are cointegrated in both two data sets. Further, the statistics,  $F_c$ , presents at least one of the two parameters is not zero. Thus, all evidence above indicates that not only long-run relationship between equity index return and index futures return for MSCI Taiwan index and Hang-Seng index, but also a threshold effect within this long-term relationship. In Table 3, we also examine if an asymmetric effect exists in the autoregressive threshold process of the residuals. The null hypothesis is set





as  $H_0: \hat{\rho}_1 = \hat{\rho}_2$ . The asymmetry exists if the null is rejected, or there is no asymmetry in the threshold autoregressive process of the residuals. The outcomes of asymmetry test in Table 3 (denoted as  $F_a$ ) are quite diverse between Taiwanese and Hong Kong markets. Generally, the null hypothesis ( $H_0: \hat{\rho}_1 = \hat{\rho}_2$ ) cannot be rejected in MSCI Taiwan index and Hang-Seng index. In other words, an asymmetric threshold cointegration model fits well for the long-run relationship between Hang-Seng index returns and its index futures returns, but the long-term relationship between MSCI Taiwan index return and its index futures return is symmetric.

[Insert Table 3 about here]

[Insert Table 4 about here]

The second non-linear cointegration model is based on a Markov-switching process. Table 4 reports the parameter estimates using the maximum likelihood method for state 1 of the lower-risk premium state and state 2 of the higher-risk premium state according to the results of estimated volatilities. On average, the market risk of Hang-Seng index (36.9241 and 48.1003 for low and high risk states, respectively) and its futures tends to be higher than the one of MSCI equity index (5.7092 and 20.6503). The risk premium can be attributed to risk transfer from hedgers in the spot market to traders in the

futures market. That is, the hedgers holding spots sell futures positions at a price below expected future spot prices to induce speculators to take long positions in the futures. Consequently, the lower premium is the state for the smaller risk to be transferred from hedgers to speculators. All parameter estimates of MSCI Taiwan index and Hang-Seng index in Table 4 are quite significant in both state 1 and state 2. In addition, the likelihood ratio test<sup>6</sup> indicates that the parameters with regime shifts are significantly different from those without regime shifts in the pair of MSCI Taiwan index.

To both MSCI Taiwan index and Hang-Seng index, the estimated parameters of  $p_{11}$  are higher than those of  $p_{22}$ . This suggests the possibility that the cointegration system remains in state 1 once the system enters state 1 is higher than the possibility that the cointegration system

<sup>6</sup> The transition probabilities under the no-regime-shift hypothesis are unidentified. Therefore, the standard asymptotical distribution for the usual likelihood ratio test cannot apply to our cases. Davies (1987) proposes a modified upper bound on the significance level of the likelihood ratio test allowing for nuisance parameters, which is given as  $\Pr(\chi_s^2 > M) + M^{s/2} [e^{M/2} \Gamma(s/2)]^{-1}$ , where  $\Gamma(\cdot)$  is the gamma function,  $M$  is the likelihood ratio, and  $s$  is the number of nuisance parameters. Hence, Davies' modified upper bound on the LR test is adopted in this paper.



remains in state 2 once the cointegration system enters state 2. On the other hand, the prediction power is more accurate in state 1 than in state 2 because state 2 is associated with high volatility. From the results of LR test, the results of the two data sets also provide a direct conclusion that cointegration model with regimes is more appropriate than the simple cointegration model.

From Table 2 to Table 4, the evidence generally show that the equity index returns are cointegrated with corresponding index futures returns based on various cointegration model. A comparison based on mean square error (MSE) between the three cointegration models (i.e., simple, regime-shifting, and threshold cointegration models) is presented in Table 5. The evidence of MSE in Table 5 show that regime switching cointegration model captures the long-run relationship between the spot and futures prices better than the cointegration without regime switching and threshold cointegration model, both in the two sets of equity indices. This suggests that the performance of the cointegration model with regime shift is better than the other two models.

[Insert Table 5 about here]

[Insert Table 6 about here]

The smoothed Markov switching probabilities of the two states for each of

the two indices are shown in Figures 1 and 2. Following the decision rule,  $Pr(z_t = t | \Omega, i = 1 \text{ or } 2) \geq 0.5$ <sup>7</sup>, the relationship between the spot and futures markets in the MSCI Taiwan tend to stay at the lower risk premium state. However, it is obscure for Hang-Seng index from Figure 2. Table 6 indicates that 74.70% of data points in MSCI Taiwan are in state 1. The high occurrence for state 2 in Figure 1 suggests that the higher risk premium evoked by the international financial turmoil in 2004 to 2005 and the subordinated-debt crisis in 2007-2008 is more in demand. Because of the time-lag effect, the model captures the impacts of the international financial turmoil on the MSCI Taiwan up until the second and third quarter of 2006, but the impact of subordinated-debt crisis is repaid and direct. From the aspect of Hang-Seng index, it seems that Hong Kong equity market was not affected by international financial turmoil happened around 2004 to 2005.

[Insert Table 6 about here]

## 5. CONCLUSION

Since the seminal paper of Engle and Granger(1987), using cointegration

<sup>7</sup> This decision rule was used widely in most of regime-switching studies. See Hamilton(1989, 1990), Engle and Hamilton(1990), Hamilton and Susmel(1994), and Hall, Pasaradakis, and Sola(1997), etc.



methods to estimate long-term relationships among economic variables has become pervasive, but the usual cointegration estimation methods do not take different economies into account and assume stable long-term relationships among cointegrated variables. Thus, the phenomena of changing economies resulting in cointegration parameter changes cannot be detected using the standard cointegration estimation methods. In order to resolve this dilemma, the non-linear cointegration method considering both cointegration and changing natures is needed. In this study, we apply two non-linear cointegration models including regime switching method and asymmetric threshold model to investigate the cointegration (long-term) relationship between spot and futures price changes. The MSCI Taiwan index, Hang-Seng index, and their corresponding futures price are applied in this study. Using Hansen (1992) and Gregory and Hansen (1996) tests, we find that there is a cointegration relationship with possible regime shifts between the spot prices and futures prices, and suggested break points are also estimated.

The cointegration tests reveal that the spot and futures prices are cointegrated. And the evidence shows that the mean square errors of the non-linear cointegration models are smaller than the

original model. Furthermore, the model with regime shift provides the best performance among the three cointegration models. We also demonstrate that the cointegration with regime shift model not only captures the long-term relationship among variables but also the structural change in the economy would which indeed affects the investors' behavior. The results also show that the risk premium in state 2 of MSCI Taiwan index and Hang-Seng index are generally higher than the ones in state 1 when the economic shocks occur. That is, the market requests a higher risk premium during the financial turmoil period. Furthermore, the model would be helpful for discovering that index price when structural changes take place.

## Appendix A

According to Eq. (1), logarithms of  $S_t$  and  $S_{t+1}$  can be rewritten as

$$S_{t+1} = \begin{cases} S_t + u_0 + \sigma_0 \eta_{t+1}, & \text{if } Z_{t+1} = 0 \\ S_t + u_1 + \sigma_1 \eta_{t+1}, & \text{if } Z_{t+1} = 1 \end{cases}$$

where  $S_t$  and  $S_{t+1}$  are the logarithms of  $S_t$  and  $S_{t+1}$ . At time  $t$ , investors' expectation of  $S_{t+1}$  will be projected on the futures price,  $F_t$ . Thus, the expectation of  $S_{t+1}$  can be seen as the futures price plus a drift as shown in Eq. (2) where  $\lambda$  represents the risk premium includes the term,  $u_0 + \sigma_0 \eta_{t+1}$ . Also, the relationship between spot price and futures price can be



confirmed in Eq. (2). In this paper, two different states of economy and the log-normal distribution of  $S_t$  are assumed. It is easy to obtain Eq. (1) by substituting  $\alpha_i = \frac{\lambda}{R_i}$ ,  $\beta_i = \frac{1}{R_i}$ ,  $R_i = \exp\left(u_i + \frac{1}{2}\sigma_i^2\right)$  into Eq. (3). Thus,

$$S_t = \frac{\lambda}{R_i} + \frac{1}{R_i} F_t$$

$$= \exp\left[-\left(u_i + \frac{1}{2}\sigma_i^2\right)\right] (\lambda + F_t)$$

Thus,

$$\ln S_t = -\left(u_i + \frac{1}{2}\sigma_i^2\right) + \ln(\lambda + F_t)$$

According to the assumption,

$$s_t = -\left(u_i + \frac{1}{2}\sigma_i^2\right) + s_{t+1}$$

So,

$$s_{t+1} = s_t + u_i + \frac{1}{2}\sigma_i^2$$

Thus, Eq. (1) is assumed based on two hypothetical economy states.

## REFERENCES

- Antoniou, A. and P. Holmes (1996) Futures market efficiency, the unbiasedness hypothesis and variance-bounds tests: the case of the FTSE-100 futures contract, *Bulletin of Economic Research*, 48, 115-128.
- Balke, N. S. and T. B. Fomby (1997) Threshold cointegration. *International Economic Review* 38, 627-366.
- Baum, C.F., J. T. Barkoulas, M. Caglayan (2001) Nonlinear adjustment to purchasing power parity in the post-Bretton Woods era. *Journal of International Money and Finance* 20, 379-399.
- Bhatt, S. and N. Cakici (1990) Premiums on stock index futures: some evidence, *Journal of Futures Markets*, 10, 367-375.
- Brenner, R.J. and K.F. Kroner (1995) Arbitrage, cointegration, and testing the unbiasedness hypothesis in financial markets, *Journal of Financial and Quantitative Analysis*, 30, 23-42.
- Chan, K. and Y.P. Chung (1993) Intraday relationships among index arbitrage, spot and futures price volatility, and spot market volume: a transactions data test, *Journal of Banking and Finance*, 17, 663-687.
- Chen, L., M. Finney, and K. S. Lai (2005) A threshold cointegration analysis of asymmetric price transmission from crude oil to gasoline prices, *Economics Letters* 89, 233-239.
- Chen, S. and J. Lin (2000) Modelling business cycles in Taiwan with time-varying Markov-switching models, *Academia Economic Papers*, 28, 17-42.
- Cheung, Y.-W. & Erlandsson, U. G. (2005) Exchange rates and Markov switching dynamics. *Journal of Business &*



- Economic Statistics*, 23, pp.314-320.
- Chowdhury, A.R. (1991) Futures market efficiency: evidence from cointegration tests, *Journal of Futures Markets*, 11, 577-589.
- Davies, R. B. (1987) Hypothesis testing when a nuisance parameter is present only under the alternative, *Biometrika*, 74, 33-43.
- Enders, W. and Siklos, P. L. (2001) Cointegration and threshold adjustment. *Journal of Business and Economic Statistics*, 19, 166-176.
- Engel, R.F. and C.W.J. Granger (1987) Cointegration and error correction: representation, estimation, and testing, *Econometrica*, 55, 251-276.
- Engle, C. and J. Hamilton (1990) Long swings in the dollar: are they the data and do markets know it? *American Economic Review*, 80, 689-713.
- Evans, M.D.D. and K.K. Lewis (1995) Do long-term swings in the dollar affect estimates of the risk premia, *Review of Financial Studies*, 8, 709-742.
- Fama, E. and K.R. French (1987) Commodity futures prices: some evidence on forecast power, premiums, and the theory of storage, *Journal of Business*, 60, 55-73.
- Figlewski, S. (1984) Explaining the early discounts on stock index futures: the case for disequilibrium, *Financial Analysts Journal*, 40, 43-47.
- Filardo, A. J. (1994) Business-Cycle phases and their transitional dynamics, *Journal of Business and Economic Statistics*, 12, 299-308.
- Ghosh, A. and R. Clayton (1996) Hedging with international stock index futures: an inter temporal error correction model, *Journal of Financial Research*, 19, 477-491.
- Gregory, A.W. and B.E. Hansen (1996) Residual-based tests for cointegration in models with regime shift, *Journal of Econometrics*, 70, 99-126.
- Hall, S.G., Z. Psaradakis, and M. Sola (1997) Cointegration and changes in regime: the Japanese consumption function, *Journal of Applied Econometrics*, 12, 151-168.
- Hamilton, J.D. (1989) A new approach to the economic analysis of nonstationary time series and the business cycle, *Econometrica*, 57, 357-384.
- Hamilton, J.D. (1990) Analysis of time series subject to changes in regime, *Journal of Econometrics*, 45, 39-70.
- Hamilton, J.D. (1991) A quasi-Bayesian approach to estimating parameters for mixtures of normal distributions, *Journal of Business and Economic Statistics*, 9, 27-39.
- Hamilton, J.D. and R. Susmel (1994)



- Autoregressive conditional heteroskedasticity and Changes in regime, *Journal of Econometrics*, 64, 307-333.
- Hansen, B.E. (1992) Tests for parameter instability in regressions with I(1) processes, *Journal of Business and Economic Statistics*, 10, 321-335.
- Kasuya, M. (2005) Regime-switching approach to monetary policy effects, *Applied Economics*, 37, 307-326.
- Koutmos, G. and M. Tucker (1996) Temporal relationships and dynamic interactions between spot and futures stock markets, *Journal of Futures Markets*, 16, 55-69.
- Kroner, K.F. and J. Sultan (1993) Time varying distributions and dynamic hedging with foreign currency futures, *Journal of Financial and Quantitative Analysis*, 28, 535-551.
- Kuo, C. J. and S. L. Lu (2005) Taiwan's financial holding companies: an empirical investigation based on Markov regime-switching model, *Applied Economics*, 37, 593-605.
- Lai, K.S. and M. Lai (1991) A cointegration test for market efficiency. *Journal of Futures Markets*, 11, 567-575.
- Lim, K. G. (1992) Arbitrage and price behavior of the Nikkei stock index futures, *Journal of Futures Markets*, 12, 151-161.
- Martens, M., P. Kofman, T. F. Vorst (1998) A threshold error-correction model for intraday futures and index returns. *Journal of Applied Econometrics* 13, 245-263.
- Ng, S. and Perron, P. (2001) Lag length selection and the construction of unit root tests with good size and power. *Econometrica*, 69, 1519-1554.
- Obstfeld, M. and A. M. Taylor (1997) Nonlinear aspects of goods market arbitrage and adjustment: Heckscher's commodity points revisited. *Journal of the Japanese and International Economies* 11, 441-479.
- Park, T.H. and L.N. Switzer (1995) Time-varying distributions and the optimal hedge ratios for stock index futures, *Applied Financial Economics*, 5, 131-137.
- Phillips, P.C.B. (1987) Time series regression with a unit root, *Econometrica*, 55, 277-301.
- Phillips, P.C.B. and B.E. Hansen (1990) Statistical inference in instrumental variables regression with I(1) processes, *Review of Economic Studies*, 57, 99-125.
- Psaradakis, Z. & Spagnolo, N. (2003) On the Determination of the Number of Regimes in Markov-Switching Autoregressive Models. *Journal of Time Series Analysis*, 24, 237-252.



- Saunders, Jr. E.M. and A. Mahajan (1988)  
An empirical examination of composite stock index futures pricing, *Journal of Futures Markets*, 8, 211-228.
- Smith, K. L. (1999) Major world equity market interdependence a decade after the 1987 Crash: evidence from cross spectral analysis, *Journal of Business Finance and Accounting*, 26, 365-392.
- Smith, A., Naik, P. A. and Tsai, C. L. (2006) Markov-switching model selection using Kullback–Leibler divergence. *Journal of Econometrics*, 134, 553-577.
- Taylor, A.M. (2001) Potential pitfalls for the purchasing-power-parity puzzle? Sampling and specification biases in mean-reversion tests of the law of one price. *Econometrica* 69, 473–498.
- Wahab, M. and M. Lashgari (1993) Price dynamics and error correction in stock index and stock index futures markets: a cointegration approach, *Journal of Futures Markets*, 13, 711-742.
- Willis, L. J. (2003) Implications of structural changes in the U.S. economy for pricing behavior and inflation dynamics, *Economic Review-Federal Bank of Kansas City*, 1th Quarter, 88, 5-27.



## Tables

Table 1 Unit Root Tests

<b>Panel A: Original Series</b>					
		<b>ADF test</b>		<b>Phillips-Perron Test</b>	
		<i>Equity index</i>	<i>Futures index</i>	<i>Equity index</i>	<i>Futures index</i>
MSCI Taiwan		-0.9019	-0.6139	-0.8979	-0.6169
Hang-Seng index		-0.9504	0.7487	-0.9954	-0.7958

<b>Panel B: First Differenced Series</b>					
		<b>ADF test</b>		<b>Phillips-Perron Test</b>	
		<i>Equity returns</i>	<i>Futures returns</i>	<i>Equity returns</i>	<i>Futures returns</i>
MSCI	Taiwan	-39.6460**	-46.9105**	-39.6657**	-47.0296**
Hang-Seng index		-46.2147**	-65.2587**	-45.6897**	-67.0258**

(a) The null hypothesis is that the series has a unit root

(b) (\*\*) denotes that the test statistic is significantly different from zero at a 10% (5%) level.

Table 2 Stability Tests

		<b>GH Test</b>		
		<i>ADF</i>	$Z_{\alpha}$	$Z_t$
MSCI	Taiwan	-4.8572**	-57.7687**	-5.5676**
	index	[0.26]	[0.26]	[0.26]
Hang-Seng index		-5.0292	-62.6520	-7.5109
		[0.34]	[0.34]	[0.34]

Note: 1. (\*\*) denotes that the test statistic is significantly different from zero at a 10% (5%) level.

2. Numbers in brackets are estimated breakpoints.





Table 3 The results of asymmetric cointegration test

	MSCI Taiwan index	Hang-Seng index
$\hat{\rho}_1$	-0.0781**	-0.0941**
$\hat{\rho}_2$	-0.0621**	-0.0307*
$F_c$	3.9457 [0.0000]	4.0185 [0.0000]
$F_a$	1.1036 [0.3024]	3.6732 [0.0000]
AIC	103.4503	67.9849
SBIC	110.5839	72.5097

Note: 1. According to Enders and Siklos (2001), the simple threshold cointegration model is a threshold autoregressive process in the residual sequence obtained from Eq. (4) with no state. The process can be shown as

$$\Delta u_t = I_t \rho_1 u_{t-1} + (1 - I_t) \rho_2 u_{t-1} + \sum_{i=1}^k \gamma_i \Delta u_{t-i} + \varepsilon_t$$

Where  $\varepsilon_t$  is a white-noise disturbance. And  $I_t$  is an indicator for  $I_t = 1$  if  $u_{t-1} \geq \tau$  and  $I_t = 0$  if  $u_{t-1} < \tau$ , where  $\tau$  is the unknown threshold value.

2.  $F_c$  and  $F_a$  indicate the  $F$ -statistics for the null hypothesis of no cointegration and symmetry. In fact,  $F_a$  includes a  $F$  test in the volatility between the two parameters and a mean-value test of the two parameters. For convenience, the symbol,  $F_a$ , is used here.

3. The threshold value ( $\tau$ ) of the threshold autoregressive model above is identified based on the AIC and SBIC. They are -0.0295 and 0.0972 for MSCI Taiwan index and Hang-Seng index, respectively.

4. We follow the procedure of Ng and Perron (2001) to set lag-length ( $k$ ) in the model above as zero.



Table 4 Parameter Estimates for Regime-Shift Cointegration Regressions

Parameter	MSCI Taiwan Index		Hang-Seng Index	
	State1	State2	State1	State2
$\alpha$	-20.7487 (0.000)	57.0255 (0.000)	64.2581 (0.009)	185.4571 (0.000)
$\beta$	0.9331 (0.000)	0.8239 (0.000)	0.9241 (0.000)	0.8954 (0.000)
$\sigma$	5.7092 (0.000)	20.6503 (0.000)	36.9241 (0.002)	48.1003 (0.001)
$p$	0.9977 (0.000)	0.9926 (0.000)	0.9219 (0.000)	0.9015 (0.001)
<b>Log Likelihood Value</b>				
With Regimes	-6050.5494		-5125.6584	
No Regimes	-7422.2743		-8656.9103	
LR-test	2743.4499 (0.0000)		-1523.1874 (0.0000)	

Note: Numbers in parentheses are p-values for the null of zero hypotheses.

Table 5 Cointegration Tests and Mean Square Error

		Mean Square Error		Threshold cointegration
		No Regime Shifts	Regime Shifts	
MSCI	Taiwan	311.8945	136.2761	259.8963
Hang-Seng index		368.5972	184.9813	289.8102

Note: 1. \*(\*\*) denotes that the null hypothesis of no cointegration is rejected at a 10% (5%) level.

2. The threshold cointegration model used in this paper is based on Enders and Siklos (2001). According to the spirit of their model, the threshold cointegration is simply a threshold autoregressive model in the residual sequence.



Table 6 Number of Cases for Smoothed Probabilities Larger Than 0.5

		<i>State 1</i>	<i>State 2</i>	<i>Total</i>
MSCI	Taiwan	1393	438	1831
index		[0.7607]	[0.2392]	
Hang-Seng		1324	451	1775
index		[0.7459]	[0.2541]	

Note: Numbers in brackets are percentages in each state with respect to total number of observations.



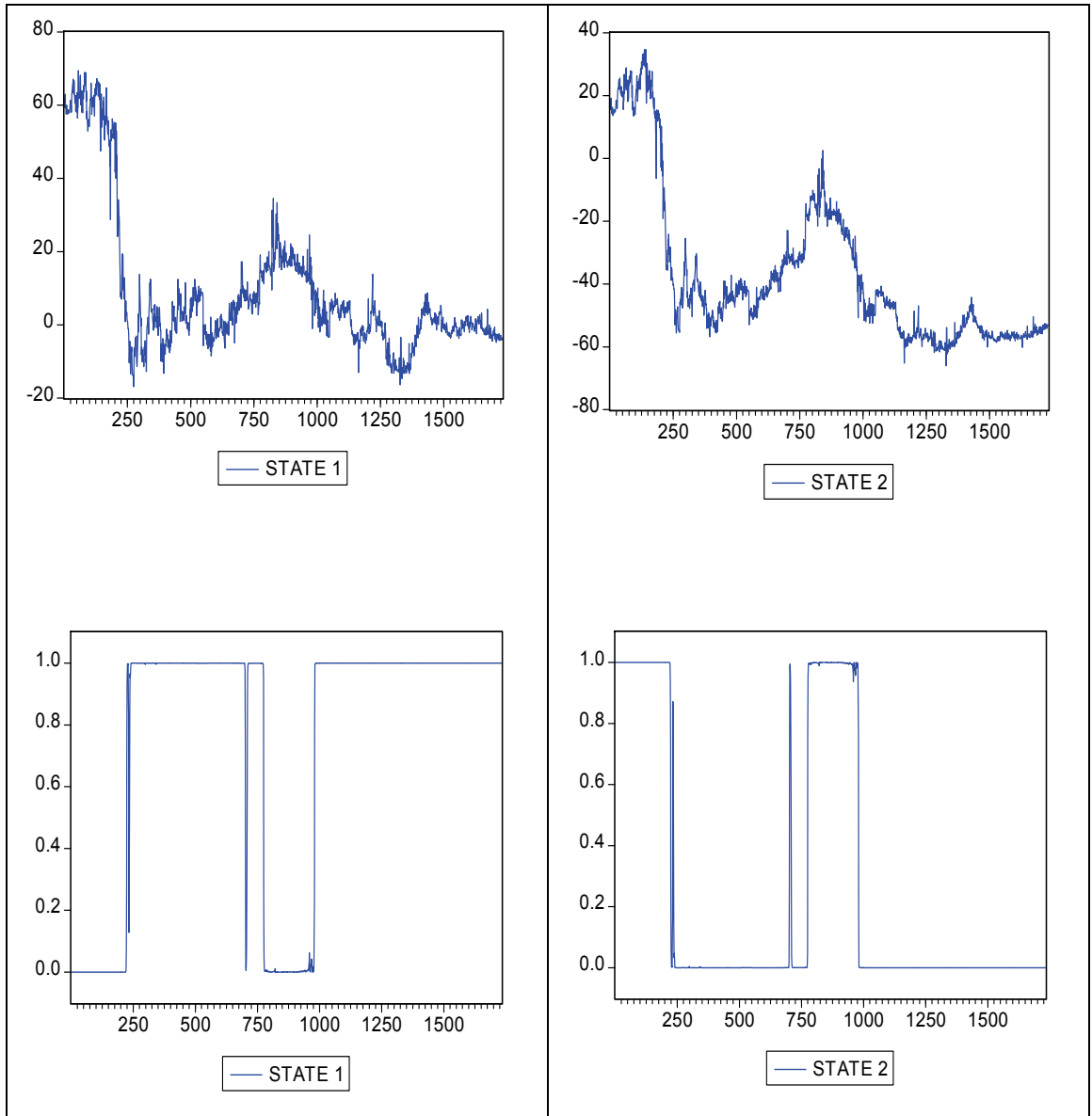


Figure 1: The residual plot and smoothed probability for the MSCI Taiwan index



