Chaos Synchronization of Rigid Body Motions

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Abstract

A new strategy to achieve chaos synchronization of rigid body motions via variable strength linear balanced feedback control is proposed. The proposed strategy gives the criteria of chaos synchronization for two identical chaotic rigid body systems. Based on Lyapunov stability theory and extreme approach, the variable strength gains are derived in term of the states of the drive system. Furthermore, an adaptive control scheme is proposed for chaos synchronization when the parametric variations of the response system are uncertain. The feasibility and effectiveness of the proposed synchronization scheme are verified via numerical simulations.

Keywords: Chaos synchronization; Rigid body motions; Variable strength linear balanced feedback control, Adaptive control.

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剛體運動系統之渾沌同步

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摘　要

本文是利用變強度均值化線性回授控制方法對剛體渾沌運動系統作渾沌同步控制。變強度均值化線性回授控制方法，是依據是依據李雅普若夫(Lyapunov)穩定定理及極值法則來設計均值化線性回授控制控制器，並根據追蹤狀態函數值來調變增益值強度。另外，進一步利用變強度增益值來設計適應控制律，使用這些控制方法，我們可以設計渾沌控制器，使得兩個渾沌剛體系統可以做同步運動。我們也以數值模擬方法來驗證控制器的效果。

關鍵詞：渾沌同步、剛體運動、變強度均值化線性回授控制、適應性控制。

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1.Introduction

In gyroscopic dynamics, chaotic attitude motion of a rigid body has received a great deal of interest among scientists. In 1981, Leipnik and Newton [1] found double strange attractors in rigid body motion with linear feedback control. The system discussed below involves three quadratic differential equations, modified from Euler's rigid body equations by the addition of linear feedback. In 2004, Chen and Lee [2] presented 2-scroll chaotic attractors in a rigid body system by applying linear feedback with certain gains and transferred this system into the famous Lorenz and Chen chaotic systems. In 2007, Chen [3] displayed a 4-scroll chaotic attractor in rigid body motions by transferring Euler's rigid body system into the corresponding Liu's chaotic system [4]. From above studies, it has found that Euler equations of a rigid body motion not only are an important three-dimensional autonomous system in classical mechanics but also the system exhibits complex chaotic behaviors such as famous Lorenz, Lü, Chen and Liu's chaotic systems by appropriate choice the feedback gains.

In recent years, synchronization in chaotic dynamic system is a very interesting problem and has been widely studied [1-19]. Synchronization means that the state variables of a response system approach eventually to that of a drive system. Since Pecora and Carrol [5] proposed the PC method for synchronizing two identical chaotic systems, various methods have been presented for the synchronization of chaotic systems such as linear feedback control [6], backstepping design [7], active control [8], nonlinear control [9] and adaptive control [10-16], etc. Furthermore, a novel synchronization control, named linear balanced feedback control, has been proposed based on linear feedback scheme and extreme approach [17-18]. The advantage of this method is that the roughly balanced feedback gains of the system can be obtained analytically and the convergent rate of state error dynamics is roughly balanced with respect to each state error. In this control scheme, balanced feedback gains are time-invariant and conservative owing to overestimating the upper bounds of the trajectory of a chaotic system in advance. Hence, a modified controller for a chaos synchronization system based on state-varying feedback gains scheme have been adopted.

In this paper, a new strategy to achieve chaos synchronization of rigid body motions via variable strength linear balanced feedback (VSLBF) control is proposed. In accordance with the result of the analysis, we use the Lyapunov approach to derive an updating law for the estimation of the unknown parameters when the parameters of the drive system are different with those of the response system. The feasibility and effectiveness of the proposed chaos synchronization schemes are demonstrated via numerical simulations.

2.A rigid body with chaotic dynamics

The Euler equations for motion of a rigid body with principle axes at the center of mass are

$$\begin{aligned} I_1\dot{\omega}_1 &= (I_2 - I_3)\omega_2\omega_3 + G_1, \\ I_2\dot{\omega}_2 &= (I_3 - I_1)\omega_3\omega_1 + G_2, \\ I_3\dot{\omega}_3 &= (I_1 - I_2)\omega_1\omega_2 + G_3, \end{aligned} \qquad (1)$$

where I_1, I_2, I_3 are the principle moments of inertia with respect to body axes, $\omega_1, \omega_2, \omega_3$ are the angular velocities about principle axes fixed at the center of mass and G_1, G_2, G_3 are three linear feedback control torque. Let torque feedback matrix $\mathbf{G} = \mathbf{H}\boldsymbol{\omega}$, where $\mathbf{G} = [G_1, G_2, G_3]^{\mathrm{T}}$, $\mathbf{H} = diag\{h_{11}, h_{22}, h_{33}\}$, $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^{\mathrm{T}}$. By a change of coordinates $\mathbf{x} = T(\boldsymbol{\omega})$ [1], the system (1) in $\mathbf{x} = [x, y, z]^T$ is

$$\begin{aligned} \dot{x} &= ax + d_1 yz, \\ \dot{y} &= by + d_2 xz, \\ \dot{z} &= cz + d_3 xy, \end{aligned} \qquad (2)$$

where

$\mathbf{H} = diag\{I_1 a, I_2 b, I_3 c\}$,

$T(\boldsymbol{\omega}) = [\lambda_1\omega_1, \lambda_2\omega_2, \lambda_3\omega_3]^{\mathrm{T}}$, and

$\lambda_1^2 = J_3 J_1 / (d_2 d_3 I_2 I_3)$, $J_1 = (I_1 - I_2)$

$\lambda_2^2 = J_1 J_2 / (d_1 d_3 I_1 I_3)$, $J_2 = (I_2 - I_3)$

$\lambda_3^2 = J_2 J_3 / (d_1 d_2 I_1 I_2)$, $J_3 = (I_3 - I_1)$

According to Liu and Chen [4], the necessary conditions for the system given in (2) to exhibit chaos are $d_1 < 0, d_2 > 0, d_3 > 0$ and $b < 0, c < 0$, $0 < \mathrm{a} < \text{-}(\mathrm{b} + \mathrm{c})$. Under such conditions, the five equilibrium points of this system are unstable [19]. The five equilibrium points are $S_0(0,0,0)$, $S_1(\bar{x}, \bar{y}, \bar{z})$, $S_2(-\bar{x}, -\bar{y}, \bar{z})$, $S_3(-\bar{x}, \bar{y}, -\bar{z})$ and $S_4(\bar{x}, -\bar{y}, -\bar{z})$, where $\bar{x} = \sqrt{bc/(d_2 d_3)}$, $\bar{y} = \sqrt{ca/(d_3 d_1)}$, $\bar{z} = \sqrt{ab/(d_1 d_2)}$.

Without loss of generality, we choose $I_1 = 2I_0, I_2 = I_0, I_3 = 3I_0$ ($I_3 > I_1 > I_2$), Euler's eqs. are

$\dot{\omega}_1 = -\omega_2\omega_3 + (h_{11}/I_1)\omega_1$,

$\dot{\omega}_2 = \omega_3\omega_1 + (h_{22}/I_2)\omega_2$,

$\dot{\omega}_3 = (1/3)\omega_1\omega_2 + (h_{33}/I_3)\omega_3$. By state transformation $\omega_1 = \sqrt{3}x$,

$\omega_2 = \sqrt{3}y$, $z = \omega_3$, and

$h_{11} = I_0$, $h_{22} = -10I_0$, $h_{33} = -12I_0$,

Euler's rigid body system (1) can transfer to the corresponding Liu's chaotic system (2) with the parameters $d_1 = -1, d_2 = 1, d_3 = 1$ and a, b, c.

Depending on the particular choice of system parameters $(a, b, c) = (5, -10, -3.8)$ *or* $(0.5, -10, -4)$, the chaotic system in (2) displays 2-scroll or 4-scroll chaotic attractors shown in Figs. 1 and 2, respectively.

3.Synchronization strategy by variable strength linear balanced feedback control

In this section, a systematic design process of synchronization of two identical chaotic rigid body systems is provided via VSLBF control.

Consider a chaotic system in the form of

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{F}(\mathbf{x}) \quad (3)$$

From the linear feedback approach, the controlled response system is given by

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + \mathbf{F}(\mathbf{y}) - \mathbf{K}(\mathbf{y} - \mathbf{x}) \quad (4)$$

where $\mathbf{x}, \mathbf{y} \in R^n$ are the state vectors, $\mathbf{A} \in R^{n\times n}$ is a constant matrix, and $\mathbf{F}(\mathbf{x})$ is a continuous nonlinear function, $\mathbf{K} = diag\{k_1, k_2, \ldots, k_n\} \in R^{n\times n}$ is a state-varying feedback matrix to be designed later. Systems (3) and (4) are considered as a drive-response system.

The dynamics of synchronization errors for rigid body systems can be expressed as

$$\dot{\mathbf{e}} = \mathbf{B}\mathbf{e} + \mathbf{F}(\mathbf{e}) \quad (5)$$

where $\mathbf{e} = \mathbf{y} - \mathbf{x}$ is the state error vector,

$$\mathbf{B} = \mathbf{A} + \mathbf{J} - \mathbf{K}, \quad \mathbf{F}(e) = \begin{bmatrix} d_1e_2e_3 \\ d_2e_1e_3 \\ d_3e_1e_2 \end{bmatrix},$$

$$\mathbf{J} = \left.\frac{\partial \mathbf{F}(\mathbf{y})}{\partial \mathbf{y}}\right|_{\mathbf{y}=\mathbf{x}} = \begin{bmatrix} 0 & d_1z_2 & d_1y_1 \\ d_2z_2 & 0 & d_2x_1 \\ d_3y_2 & d_3x_1 & 0 \end{bmatrix}.$$

The aim of synchronization is to make $\text{limit}_{t\to\infty} \|\mathbf{e}\| = 0$.

The problem of synchronization between the drive and response systems can be translated into a problem of how to realize the globally exponentially stabilization of the system (5). Here, assume that the states of the drive and response systems are measurable.

Construct a Lyapunov function

$$V(\mathbf{e}) = \mathbf{e}^{\mathbf{T}}\mathbf{P}\mathbf{e}, \quad (6)$$

where $\mathbf{P}$ is a positive definite diagonal constant matrix.

Its derivative along the trajectory of system (5) is

$$\begin{aligned} \dot{V} &= \dot{\mathbf{e}}^{\mathrm{T}}\mathbf{P}\mathbf{e} + \mathbf{e}^{\mathrm{T}}\mathbf{P}\dot{\mathbf{e}} = \mathbf{e}^{\mathrm{T}}(\mathbf{B}^{\mathrm{T}}\mathbf{P} + \mathbf{P}\mathbf{B})\mathbf{e} \\ &+ (\mathbf{F}^{\mathrm{T}}\mathbf{P}\mathbf{e} + \mathbf{e}^{\mathrm{T}}\mathbf{P}\mathbf{F}) = -\mathbf{e}^{\mathbf{T}}\mathbf{Q}\mathbf{e} \end{aligned}, \quad (7)$$

where $\mathbf{Q} \in R^{n\times n}$ is a positive definite matrix of variables $\mathbf{x}$. Assume that there exists a positive definite diagonal matrix $\mathbf{P}$ such as $\mathbf{F}^{\mathrm{T}}\mathbf{P}\mathbf{e} = \mathbf{e}^{\mathrm{T}}\mathbf{P}\mathbf{F} = 0$, i.e., in this system $d_1 = -1, d_2 = 1, d_3 = 1$ choose $p_1 = 1, p_2 = 0.5, p_3 = 0.5$ such that $\mathbf{F}^{\mathrm{T}}\mathbf{P}\mathbf{e} = (d_1 p_1 + d_2 p_2 + d_3 p_3) e_1 e_2 e_3 = 0$. In fact, most of chaotic systems, including Lorenz, Lü, Chen and four-scroll new chaotic systems [2-4] can be described by this expression.

Chaos synchronization problem is to design a state-varying feedback gain matrix $\mathbf{K}$ to make the matrix $\mathbf{Q}$ a positive definite function. Then the states of the response system and drive system are globally asymptotically synchronized. To implement balanced feedback gains, a method minimizing the sum of the feedback gains is adopted to obtain a set of roughly equal control gains. The procedure for designing control gains is described as follows:

The first step is to solve the linear feedback control gains from the positive definite matrix $\mathbf{Q}$. Assume all the principal minor determinants corresponding to the symmetric matrix $\mathbf{Q}$ as following

$$\Delta_i = |Q_{rr}| = m_i > 0, \quad r = 1,2,\ldots,i,\ i = 1,2,\ldots,n. \tag{8}$$

From (8), we obtain

$$k_i = S_i(m_1, m_2, \ldots, m_r), \quad r = 1,2,\ldots,i,\ i = 1,2,\ldots,n, \tag{9}$$

and

$$\frac{\partial k_i}{\partial m_i} \neq 0, \quad \frac{\partial k_j}{\partial m_i} = 0, \quad j = 1,2,\ldots i-1,\ i = 1,2,\ldots,n. \tag{10}$$

The second step is to minimize the sum of the control gains, i.e., $f = Min(k_1 + \ldots + k_n)$. This means that the control gains are roughly equal, i.e. balanced.

The third step is to study minima of function of specific variables. Then, write down the necessary conditions for rendering f a relative maximum or minimum as follows:

$$\frac{\partial f}{\partial m_i} = 0, \quad i = 1,2\ldots,n. \tag{11}$$

By solving (11) corresponding to (10), the extreme point $(q_1^*, q_2^*, \ldots q_n^*)$ is found.

4.Synchronization of two identical chaotic systems

Consider the rigid body chaotic system as the drive system described as

$$\begin{aligned} \dot{x}_1 &= a x_1 + d_1 y_1 z_1, \\ \dot{y}_1 &= b y_1 + d_2 x_1 z_1, \\ \dot{z}_1 &= c z_1 + d_3 x_1 y_1, \end{aligned} \tag{12}$$

The controlled response system is described as

$$\dot{x}_2 = ax_2 + d_1 y_2 z_2 - k_1(x_2 - x_1),$$
$$\dot{y}_2 = by_2 + d_2 x_2 z_2 - k_2(y_2 - y_1),$$
$$\dot{z}_2 = cz_2 + d_3 x_2 y_2 - k_3(z_2 - y_1). \quad (13)$$

Subtracting Eq. (12) from (13), we can obtain the error dynamics in the form of

$$\dot{\mathbf{e}} = \mathbf{Be} + \mathbf{F}(\mathbf{e}) \quad (14)$$

where

$$\mathbf{B} = \begin{bmatrix} a-k_1 & d_1 z_2 & d_1 y_1 \\ d_2 z_2 & b-k_2 & d_2 x_1 \\ d_3 y_2 & d_3 x_1 & c-k_3 \end{bmatrix},$$

$$\mathbf{F}(e) = \begin{bmatrix} d_1 e_2 e_3 \\ d_2 e_1 e_3 \\ d_3 e_1 e_2 \end{bmatrix}.$$

Construct a Lyapunov function

$$V = e^{\mathbf{T}}\mathbf{P}e \quad (15)$$

where $\mathbf{P} = diag\{p_1, p_2, p_3\}$ is a positive definite matrix.

The derivative of the Lyapunov function along the trajectory of system (14) is

$$\dot{V} = \dot{\mathbf{e}}^{\mathrm{T}}\mathbf{Pe} + \mathbf{e}^{\mathrm{T}}\mathbf{P}\dot{\mathbf{e}} = \mathbf{e}^{\mathrm{T}}(\mathbf{B}^{\mathrm{T}}\mathbf{P} + \mathbf{PB})\mathbf{e} + (\mathbf{F}^{\mathrm{T}}\mathbf{Pe} + \mathbf{e}^{\mathrm{T}}\mathbf{PF}) = -\mathbf{e}^{\mathbf{T}}\mathbf{Qe}, \quad (16)$$

where $\mathbf{F}^{\mathrm{T}}\mathbf{Pe} = \mathbf{e}^{\mathrm{T}}\mathbf{PF} = 0$ and

$$\mathbf{Q} = \begin{bmatrix} q_1 & n_1 & n_2 \\ n_1 & q_2 & n_3 \\ n_2 & n_3 & q_3 \end{bmatrix} = \begin{bmatrix} 2(k_1-a)p_1 & p_3 d_3 z_2 & p_2 d_2 y_1 \\ p_3 d_3 z_2 & 2(k_2-b)p_2 & p_1 d_1 x_1 \\ p_2 d_2 y_2 & p_1 d_1 x_1 & 2(k_3-c)p_3 \end{bmatrix}.$$

Obviously, to ensure that the origin of error system (14) is asymptotically stable, the matrix Q should be positive definite. By Sylvester's theorem, all principal minors of Q are strictly positive, i.e., a suitable linear feedback gain matrix K can be chosen if the following conditions hold:

$$k_1 = a + m_1/(2p_1),$$
$$k_2 = b + (m_2 + n_1^2)/(2m_1 p_2) = b + (m_2 + p_3^2 N_1^2)/(2m_1 p_2),$$
$$k_3 = c + [m_1 m_3 + (m_1 n_3 - n_1 n_2)^2 + m_2 n_2^2]/(2m_1 m_2 p_3) \le k_3^* \quad (17)$$

where m_1, m_2, m_3 are positive constant,

$$k_3^* = c + [m_1 m_3 + (m_1 p_3 N_3 + p_1 p_2 N_1 N_2)^2 + m_2 p_2^2 N_2^2]/(2m_1 m_2 p_3)$$
$$N_1 = |n_1|/p_3 = |d_3 z_1|,$$
$$N_2 = |n_2|/p_2 = |d_2 y_1|,$$
$$N_3 = |n_3|/p_1 = |d_1 x_1|.$$

Choose $k_3 = k_3^*$. To obtain balanced feedback gains, a method minimizing the sum of the feedback gains is presented as follows. The minimization of the functional $f = Min\{k_1 + k_2 + k_3\}$ is required. From (10) $\frac{\partial k_3}{\partial m_3} \neq 0$, $\frac{\partial k_j}{\partial m_3} = 0$, $j = 1,2$, we know that the necessary condition $\frac{\partial f}{\partial m_3} = 0$ fails to exist. Thus, it may happen that an extreme value is taken on at a boundary point, i.e.,

$m_3 = 2\varepsilon p_3 m_2, \varepsilon \to 0^+$.

In case study, a set of parameters of the chaotic system is defined by $d_1 = -1, d_2 = d_3 = 1$， $a > 0, b < 0, c < 0$, and assuming $p_1 = p_0, p_2 = p_3 = p_0/2$.

By solving the minimization of the functional $f = Min\{k_1 + k_2 + k_3\}$, we can obtain linear balanced feedback gains. If $f_{m_1} = 0, f_{m_2} = 0$, and $f_{m_1 m_1} > 0$, $f_{m_1 m_1} f_{m_2 m_2} > f^2_{m_1 m_2}$ at a point $p(m_1^*, m_2^*)$, then at that point *f* has a relative minimum. With

$$f(k_1, k_2, k_3) = k_1 + k_2 + k_3 = f(m_1, m_2, 0^+) \quad (18)$$

the necessary conditions $f_{m_1} = f_{m_2} = 0$ at (m_1^*, m_2^*) become

$$f_{m_1} = (8m_1^2 m_2 - (p_0^2 N_1^2 + 4m_2)(p_0^2 N_2^2 + 4m_2) + 16 p_0^2 m_1^2 N_3^2)/(16 p_0 m_1^2 m_2) = 0,$$

$$f_{m_2} = (16m_2^2 - (4p_0 m_1 N_3 + p_0^2 N_1 N_2)^2)/(16 p_0 m_1 m_2^2) = 0, \quad (19)$$

from which there follows

$$m_1^* = \sqrt{2}(N_1 + N_2) p_0 / 2 > 0,$$

$$m_2^* = (N_1 N_2 + 2\sqrt{2} N_2 N_3 + 2\sqrt{2} N_3 N_1) p_0^2 / 4 > 0. \quad (20)$$

Thus, the corresponding minimum sum of control gains is

$$Min\{k_1 + k_2 + k_3\} = (a + b + c') + \sqrt{2}(N_1 + N_2)/2 + 2N_3,$$ i.e.,

$$k_1 = a + \sqrt{2}(N_1 + N_2)/4,$$

$$k_2 = b + \sqrt{2}N_1/4 + N_3,$$

$$k_3 = c' + \sqrt{2}N_2/4 + N_3 \quad (21)$$

where $c' = c + \varepsilon, \varepsilon \to 0^+$.

The above state-varying feedback gains are devised to ensure the global synchronization for rigid body chaotic systems. Furthermore, the convergent rate of state error dynamics can be improved by the addition of gain tuning parameter μ and Eq. (21) can be rewritten as

$$k_1^* = a + \sqrt{2}(N_1 + N_2)/4 + \mu,$$

$$k_2^* = b + \sqrt{2}N_1/4 + N_3 + \mu,$$

$$k_3^* = c' + \sqrt{2}N_2/4 + N_3 + \mu. \quad (22)$$

5.Adaptive synchronization

In Section 3, the varying parameters k_i are determined based on N_1, N_2, N_3, (*i.e.*, $|d_3 z_1|, |d_2 y_1|, |d_3 x_1|$) and μ, and the perfect knowledge of the system parameters a, b, and c. However, in many practical situations, some of the system parameters are unknown. Therefore, an adaptive synchronization scheme is presented to identify the uncertain parameters and assure the robustness of the proposed controller.

From (4), (13) and (22), the controlled response system with parametric variations can be rewritten as

$$\dot{\hat{\mathbf{y}}} = \mathbf{A}\hat{\mathbf{y}} + e_a f_a(\hat{\mathbf{y}}) + e_b f_b(\hat{\mathbf{y}}) + e_c f_c(\hat{\mathbf{y}}) + \mathbf{F}(\hat{\mathbf{y}}) - (\mathbf{K}^* + \mathbf{K}_e^*)(\hat{\mathbf{y}} - \mathbf{x}) \quad (23)$$

where $\hat{\mathbf{y}} \in R^n$ denotes the state vector of the response system with uncertain parameters, $e_a = a_h - a$, $e_b = b_h - b$, $e_c = c_h - c$ represent the parameter errors, a_h, b_h, c_h are estimated parameters, $f_a(\hat{\mathbf{y}}) = [x_2 \ 0 \ 0]^T$, $f_b(\hat{\mathbf{y}}) = [0 \ y_2 \ 0]^T$, $f_c(\hat{\mathbf{y}}) = [0 \ 0 \ z_2]^T$, $\mathbf{K}_e^* = diag\{e_a, e_b, e_c\}$ and $\mathbf{K}^* = \mathbf{K} + \mu$, $\mu = diag\{\mu_1, \mu_2, \mu_3\}$.

The dynamics of adaptive synchronization errors can be expressed as

$$\dot{\mathbf{s}} = (\mathbf{B}^* - \mathbf{K}_e^*)\mathbf{s} + \mathbf{G}\mathbf{s}_p + \mathbf{F}(\mathbf{s}) \quad (24)$$

where $\mathbf{s} = \hat{\mathbf{y}} - \mathbf{x} = [s_1, s_2, s_3]^T$, $\mathbf{s}_p = [e_a, e_b, e_c]^T$, $\mathbf{B}^* = \mathbf{A} + \mathbf{J} - \mathbf{K}^*$, $\mathbf{J} = \left.\frac{\partial \mathbf{F}(\hat{\mathbf{y}})}{\partial \hat{\mathbf{y}}}\right|_{\hat{\mathbf{y}}=\mathbf{x}}$ is the Jacobian matrix evaluated at $\hat{\mathbf{y}} = \mathbf{x}$, and $\mathbf{G} = [\ f_a(\hat{\mathbf{y}}),\ f_b(\hat{\mathbf{y}}),\ f_c(\hat{\mathbf{y}})]$.

Choose the Lyapunov function candidate

$$V_1(\mathbf{s}, \mathbf{s}_p) = \mathbf{s}^T\mathbf{P}\mathbf{s} + \mathbf{s}_p^T\mathbf{R}\mathbf{s}_p , \quad (25)$$

where $\mathbf{P} = diag\{p_1, p_2, p_3\}$ and $\mathbf{R} = diag\{r_1, r_2, r_3\}$ are positive definite diagonal constant matrices. The derivative of the Lyapunov function along the trajectory of system (24) is

$$\begin{aligned}\dot{V}_1(\mathbf{s}, \mathbf{s}_p) &= \dot{\mathbf{s}}^T\mathbf{P}\mathbf{s} + \mathbf{s}^T\mathbf{P}\dot{\mathbf{s}} + \dot{\mathbf{s}}_p^T\mathbf{R}\mathbf{s}_p + \mathbf{s}_p^T\mathbf{R}\dot{\mathbf{s}}_p \\ &= \mathbf{s}^T[(\mathbf{B}^* - \mathbf{K}_e^*)^T\mathbf{P} + \mathbf{P}(\mathbf{B}^* - \mathbf{K}_e^*)]\mathbf{s} \\ &\quad + (\mathbf{F}^T\mathbf{P}\mathbf{s} + \mathbf{s}^T\mathbf{P}\mathbf{F}) + 2\mathbf{s}_p^T\mathbf{G}^T\mathbf{P}\mathbf{s} + 2\mathbf{s}_p^T\mathbf{R}\dot{\mathbf{s}}_p \\ &= -\mathbf{s}^T\mathbf{Q}^*\mathbf{s} - 2\mathbf{s}^T\mathbf{K}_e^{*T}\mathbf{P}\mathbf{s} + 2\mathbf{s}_p^T(\mathbf{G}^T\mathbf{P}\mathbf{s} + \mathbf{R}\dot{\mathbf{s}}_p)\end{aligned} \quad (26)$$

where $\mathbf{F}^T\mathbf{P}\mathbf{s} = \mathbf{s}^T\mathbf{P}\mathbf{F} = 0$.

Select

$$-2\mathbf{s}^T\mathbf{K}_e^{*T}\mathbf{P}\mathbf{s} + 2\mathbf{s}_p^T(\mathbf{G}^T\mathbf{P}\mathbf{s} + \mathbf{R}\dot{\mathbf{s}}_p) = 0 , \quad (27)$$

then

$$\dot{V}_1(\mathbf{s}, \mathbf{s}_p) = -\mathbf{s}^T\mathbf{Q}^*\mathbf{s} , \quad (28)$$

where $\mathbf{Q}^*$ has the similar form in Eq. (16). $\dot{V}_1(\mathbf{s}, \mathbf{s}_p)$ is negative semi-definite function of the state error $\mathbf{s}$ and $\mathbf{s}_p$. By partial stability theory [15] the partial variables $\mathbf{s}$ in Eq. (24) are asymptotically stable about $\mathbf{s} = \mathbf{0}$, the synchronization manifold is stable. Hence the drive and response systems can be synchronized.

From (27), we can obtain the adaptation law of the form

$$\begin{aligned}\dot{e}_a &= (s_1^2 - x_2 s_1)p_1 / r_1 , \\ \dot{e}_b &= (s_2^2 - y_2 s_2)p_2 / r_2 , \\ \dot{e}_c &= (s_1^2 - z_2 s_3)p_3 / r_3\end{aligned} \quad (29)$$

6.Numerical results

The phase portraits of the system (12) corresponding to the initial states $x_1(0)=1,\ y_1(0)=1,\ z_1(0)=1$ are plotted in Fig. 1 and 2 at $(a,b,c)=$ $(5,-10,-3.8)$, $(0.5,-10,-4)$ respectively. Fig. 1 and 2 confirm that this system is characterized by two- and four-scroll chaotic attractors, respectively. The chaotic system in (12) has five unstable equilibrium points, i.e. $S_0(0,0,0)$, $S_1(\bar{x}_1,\bar{y}_1,\bar{z}_1)$, $S_2(-\bar{x}_1,-\bar{y}_1,\bar{z}_1)$, $S_3(-\bar{x}_1,\bar{y}_1,-\bar{z}_1)$ and $S_4(\bar{x}_1,-\bar{y}_1,-\bar{z}_1)$, where $(\bar{x}_1,\bar{y}_1,\bar{z}_1)=(6.1644,4.3589,7.0711)$ or $(\bar{x}_1,\bar{y}_1,\bar{z}_1)=(6.3246,1.4142,2.2361)$ for $(a,b,c)=(5,-10,-3.8)\ or\ (0.5,-10,-4)$, respectively. Since the equilibrium points of the chaotic rigid body system are unstable, a control problem arises. Accordingly, this study designs a VSLBF control scheme to drive the chaotic attractor to a nontrivial fixed point of the system. Assuming that $p_1=1, p_2=0.5$ and $p_3=0.5$ the feedback gains k_i and parameters m_i required to drive the chaos to point S_1 are found from Eqs. (21) and (20) to be $k_1=9.0411,$, $k_2=0, k_3=3.9055$ and $m_1=8.0822, m_2=57.5276, m_3=0^+$ or $k_1=1.7906, k_2=0, k_3=2.8246$ and $m_1=2.5811, m_2=17.1151, m_3=0^+$. Fig. 3 shows that the controller stabilizes the two-scroll chaotic attractor at an unstable equilibrium point S_1 for $(a,b,c)=(5,-10,-3.8)$ and an initial state of $x_2(0)=-10,$ $y_2(0)=-17,\ z_2(0)=15$ Fig. 4 shows that the controller stabilizes the four-scroll chaotic attractor at an unstable equilibrium point S_1 for $(a,b,c)=$ $(0.5,-10,-4)$ and an initial state of $x_2(0)=3,\ y_2(0)=-3,\ z_2(0)=3$. To drive the chaotic attractor to a nontrivial fixed point of the driving system, the feedback gains of the response system are designed at the fixed control gains via this proposed scheme.

The numerical simulations for synchronization of the two identical chaotic four-scroll systems are carried out as shown in Fig. 5-10. The initial states of the drive system (12) and response system (13) are $x_1(0)=1,$ $y_1(0)=1,\ z_1(0)=1$ and $x_2(0)=3,$ $y_2(0)=-3,\ z_2(0)=3$, respectively. In Fig. 5 and Fig. 7, the variable strength feedback gains $k_1,\ k_2$ and k_3 are roughly equal and vary with the states of drive system (12) at the tuning parameters of $\mu=0$ and $\mu=1$, respectively. The corresponding synchronization errors e_i, parameters m_i, and norms of controller output in Fig. 5 and Fig. 7 are shown in Fig. 6 and Fig. 8. The results show that chaos synchronization is achieved successfully. With the addition of gain tuning parameter such as $\mu = 1$, the proposed variable strength linear balanced feedback control scheme can achieves a rapid convergent rates of chaos synchronization of the two systems shown in Fig. 8.

The dynamics of adaptive synchronization errors, changing parameters and the variable strength feedback gains k_1, k_2 and k_3 are shown in Fig. 9 and Fig. 10. The initial states of the drive and response systems (12) and (23) are $x_1(0) = 2, y_1(0) = 3, z_1(0) = 2$, $x_2(0) = -6, y_2(0) = 6, z_2(0) = 4$, respectively, and the initial values of a_h, b_h, c_h are $(0, -10.5, -4.5)$, i.e., $\mathbf{s}_p = [e_a, e_b, e_c]^T = [-0.5, -0.5, -0.5]^T$. Fig. 9(a) displays dynamics of adaptive synchronization errors (s_1, s_2, s_3) in system (24) with the parameter estimation update law (29) and variable strength feedback gains k_1, k_2 and k_3 (22) shown in Fig. 10. The dynamics of parameters estimation update law are shown in Fig. 9(b)-(d). The numerical results show that adaptive synchronization is achieved successfully when the parameters of the drive system are different with those of the response system.

7.Conclusion

In this paper, a new strategy to achieve chaos synchronization of rigid body motions is proposed by using variable strength linear balanced feedback control. The proposed strategy gives a procedure to design variable strength linear balanced feedback gains for chaos synchronization of two four-scroll chaotic systems. In this method, the feedback gains of the system are roughly balanced and the convergent rate of the error dynamics can be improved by adopting a suitable gain tuning parameter μ. Additionally, based on the above-mentioned results, an adaptive control scheme is proposed for chaos synchronization when the parameters of the drive system are different with those of the response system. The feasibility and effectiveness of the synchronization schemes have been verified via numerical simulations.

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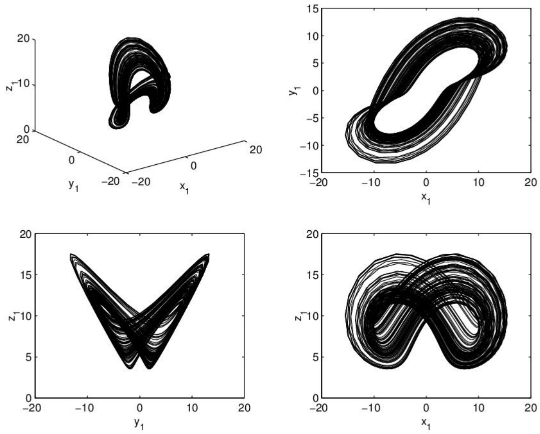

Fig.1 The two-scroll chaotic attractor of a rigid body system (2) at a = 5, b = -10, c = -3.8.

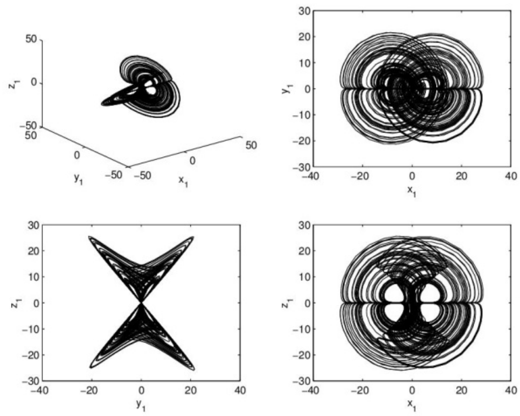

Fig.2 The four-scroll chaotic attractor of a rigid body system (2) at a = 0.5, b = -10, c = -4.

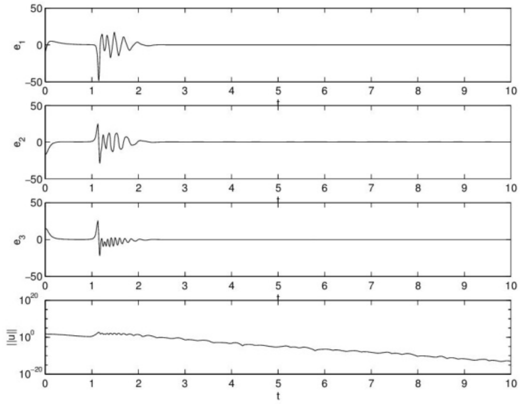

Fig.3 Variation of state errors (e_1, e_2, e_3) and norms of controller output $\|u\| = \sqrt{(k_1 e_1)^2 + (k_2 e_2)^2 + (k_3 e_3)^2}$ of system in (14) over time at, a = 5, b = -10, c = -3.8, $k_1 = 9.0411, k_2 = 0, k_3 = 3.9055$ and fixed states $S_1(\bar{x}_1, \bar{y}_1, \bar{z}_1) = (6.1644, 4.3589, 7.0711)$ in system (12).

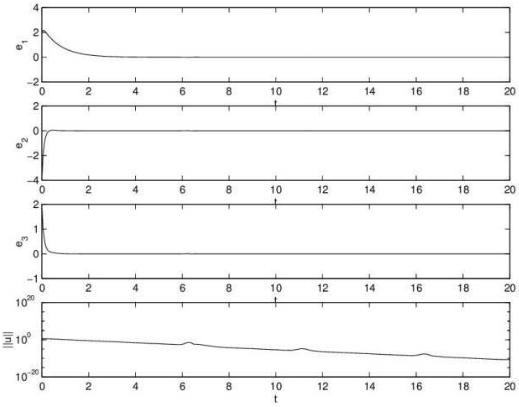

Fig.4 Variation of state errors (e_1, e_2, e_3) and norms of controller output $\|u\| = \sqrt{(k_1e_1)^2 + (k_2e_2)^2 + (k_3e_3)^2}$ of system in (14) over time at a = 0.5, b = -10, c = -4, $k_1 = 1.7906, k_2 = 0, k_3 = 2.8246$ and fixed states $S_1(\bar{x}_1, \bar{y}_1, \bar{z}_1) =$ $(6.3246, 1.4142, 2.2361)$ in system (12).

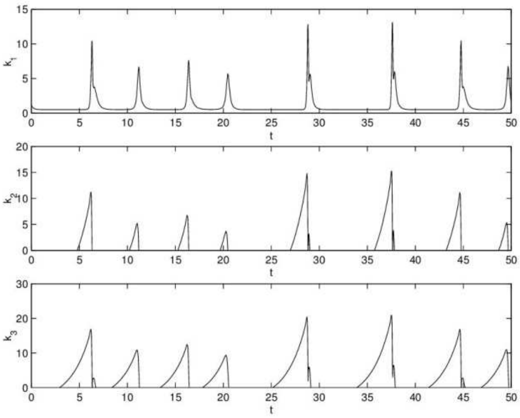

Fig.5 Variable strength feedback gains $k_1(-), k_2(—), k_3(-.)$ versus time t from Eq. (22) at the tuning parameters of $\mu = 0$.

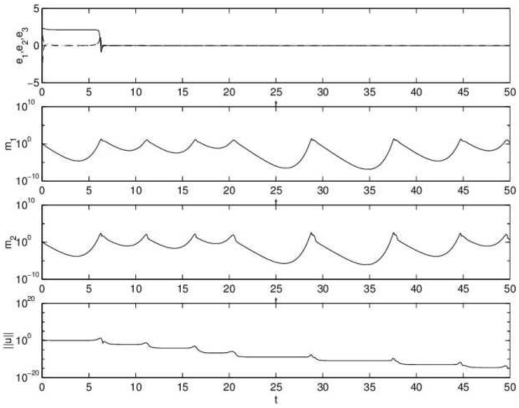

Fig.6 Dynamics of synchronization errors e_1, e_2, e_3 , parameters m_1, m_2, and norms of controller output $\|u\|$ with respect to Fig. 5.

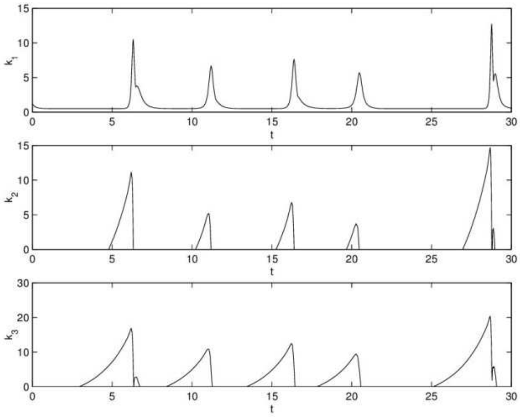

Fig.7 Variable strength feedback gains $k_1(-), k_2(—), k_3(-.)$ versus time t from Eq. (22) at the tuning parameters of $\mu = 1$.

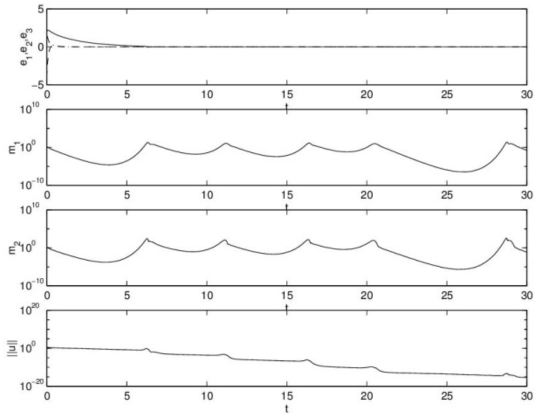

Fig.8 Dynamics of synchronization errors e_1, e_2, e_3 , parameters m_1, m_2, and norms of controller output $\|u\|$ with respect to Fig. 7.

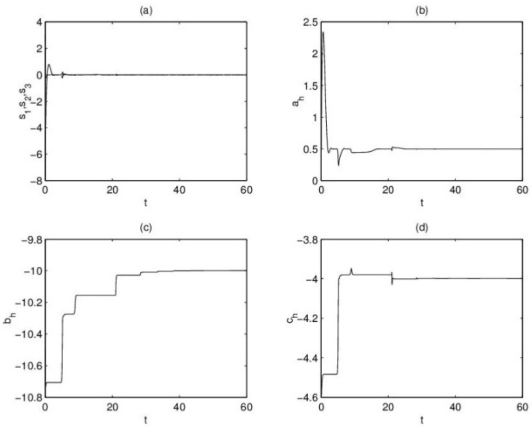

Fig.9 Dynamics of adaptive synchronization errors (s_1, s_2, s_3) for chaotic systems (12) and (23) and variations of parameters a_h, b_h, c_h of system (23) with time t at the tuning parameters of $\mu = 5$.

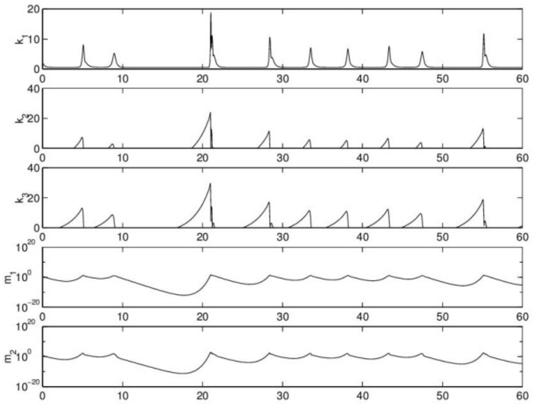

Fig.10 Dynamics of variable strength feedback gains k_1, k_2, k_3 and parameters m_1, m_2 with respect to Fig. 9.