Application of Interactive Fuzzy Multi-Objective Linear Programming to Aggregate Production Planning Decisions

Tien-Fu Liang*, Ming-Hsun Lee, Bo-Yan Zhu, Hong-Yu Huang, Min-Ting Liu, Han-Mei Hu

Abstract

This work develops an interactive fuzzy multi-objective linear programming (i-FMOLP) method for solving the multi-product and multi-time periods aggregate production planning (APP) decision problem. The imprecise multi-objective APP model designed here seeks to minimize total production costs and changes in work-force levels. The proposed i-FMOLP method provides a systematic framework that helps the decision-making process to solve multi-objective APP problems, enabling a decision maker to interactively modify the imprecise data and parameters until a set of satisfactory solutions is derived. An industrial case demonstrates the feasibility of applying the proposed i-FMOLP method to a practical APP problem.

Keywords: Aggregate Production Planning, Interactive Fuzzy Multi-Objective Linear Programming, Triangular Fuzzy Numbers, Fuzzy Sets

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互動式模糊多目標線性規劃於整體生產計劃決策之應用

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摘要

本文目的主要在於發展一互動式模糊多目標線性規劃（i-FMOLP）方法，用以求解不確定環境下涵蓋多產品、多時期之整體生產規劃（APP）決策問題。首先，本文建構一符合實務情境的不精確多目標 APP 模式，內容同步追求總生產成本及總人力水準變動率二個極小化目標；其次，本文發展一 i-FMOLP 方法之系統化架構，作為模糊多目標 APP 問題之決策程序，並提供決策者以互動化方式持續修正不精確資料和參數，直到獲得令其滿意的一組妥協解為止。最後，本文特舉一產業實際個案進行模式測試，驗證模式於實務應用之可行性，並彙總重要的管理意涵。

關鍵詞：整體生產規劃、互動式模糊多目標線性規劃、三角模糊數、模糊集

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1. Introduction

Aggregate production planning (APP) decisions are concerned with determining the quantity and timing of production for the intermediate future, often from 2 to 18 months ahead. Since Holt et al. [4] proposed the HMMS rule, APP has attracted considerable attention from both practitioners and academia. Numerous decision techniques including mathematical programming models, algorithms and heuristics have also been presented to solve APP problems [6-7].

When any of these conventional APP techniques are used, however, the goals and related parameters are generally assumed to be deterministic/crisp and only APP problems with the single goal of minimizing cost or maximizing profit can be solved. In practical APP decisions, related model inputs and environmental coefficients are generally fuzzy/imprecise because information is incomplete and/or unobtainable. Obviously, conventional deterministic decision techniques cannot solve practical APP problems in fuzzy environments.

In real-life APP decisions, the many functional areas in an organization that yield an input to the aggregate plan generally have conflicting goals regarding the use of organization's resources, and these conflicting goals are required to be solved simultaneously by the decision maker (DM) in the framework of imprecise aspiration levels [1, 8]. Solutions to multi-objective APP optimization problems benefit from assessing the imprecision of the DM's judgments.

This work aims to present a fuzzy mathematical programming approach for solving the multi-product and multi-time period APP problems with fuzzy goals and fuzzy market demand. The multi-objective linear programming (MOLP) model designed here attempts to simultaneously minimize total production costs and total rates of changes in labor levels, and considers the time value of money for each of the operating cost categories.

2. Problem formulation

2.1. Problem description and notation

The fuzzy multi-objective APP decision problem examined in this work can be described as follows. Assume that an industrial company manufactures N types of products to fulfill market demand over the planning horizon T. The goals of this APP decision are to minimize total production costs and total rate of change in labor levels in relation to inventory levels, machine capacity, labor levels, warehouse space, and the constraint on available total budget. The following notation is used.

- Index sets

n index for product type, for all $n = 1, 2, \cdots, N$

t index for planning time period, for all $t = 1, 2, \cdots, T$

- Objective functions

z_1 total production costs

z_2 total rate of change in labor levels

- Decision variables

Q_{nt} regular production volume for *n*th product in period t

O_{nt} overtime production volume for *n*th product in period t

S_{nt} subcontracting volume for *n*th product in period t

I_{nt} inventory level for *n*th product in period *t*

B_{nt} backordering volume for *n*th product in period t

H_t workers hired in period t

F_t workers laid off in period t

- Parameters

$\widetilde{D}_{nt}$ market demand for *n*th product in period t

a_{n1} regular production cost per unit for nth product in period 1

i_a escalating factor for regular production cost

b_{n1} overtime production cost per unit for nth product in period 1

i_b escalating factor for overtime production cost

c_{n1} subcontracting cost per unit for nth product in period 1

i_c escalating factor for subcontracting cost

d_{n1} inventory carrying cost per unit for nth product in period 1

i_d escalating factor for inventory carrying cost

e_{n1} backordering cost per unit for nth product in period 1

i_e escalating factor for backordering cost

k_1 cost to hire one worker in period 1

m_1 cost to layoff one worker in period 1

i_f escalating factor for hire and layoff cost

l_{nt} hours of labor per unit of nth product in period t

r_{nt} hours of machine usage per unit of nth product in period t

v_{nt} warehouse spaces per unit of nth product in period t
$\widetilde{W}_{t\,\max}$ maximum labor levels available in period t
$\widetilde{M}_{t\,\max}$ maximum machine capacity available in period t
$V_{t\,max}$ maximum warehouse space available in period t
Z available total budget

2.2. Fuzzy multi-objective linear programming model

2.2.1. Objective functions

• Minimize total production costs

$$\text{Min } \tilde{z}_1 = \sum_{n=1}^{N}\sum_{t=1}^{T}\Big[\tilde{a}_{n1}Q_{nt}(1+i_a)^t + \tilde{b}_{n1}O_{nt}(1+i_b)^t + \tilde{c}_{n1}S_{nt}(1+i_c)^t + \tilde{d}_{n1}I_{nt}(1+i_d)^t + \tilde{e}_{n1}B_{nt}(1+i_d)^t\Big] + \sum_{t=1}^{T}(\tilde{k}_1 H_t + \tilde{m}_1 F_t)(1+i_f)^t \quad (1)$$

• Minimize total rate of change in labor levels

$$\text{Min } z_2 \cong \sum_{t=1}^{T}(H_t + F_t) \quad (2)$$

2.2.2. Constraints

• Constraints on carrying inventory

$$I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} = \widetilde{D}_{nt} \quad \forall n, \forall t \quad (3)$$

• Constraints on labor levels

$$\sum_{n=1}^{N} l_{nt-1}(Q_{nt-1}+O_{nt-1}) + H_t - F_t = \sum_{n=1}^{N} l_{nt}(Q_{nt}+O_{nt}) \quad \forall t \quad (4)$$

$$\sum_{n=1}^{N} l_{nt}(Q_{nt}+O_{nt}) \leq \widetilde{W}_t \quad \forall t \tag{5}$$

• Constraints on machine capacity

$$\sum_{n=1}^{N} r_{nt}(Q_{nt}+O_{nt}) \leq \widetilde{M}_{t\,max} \quad \forall t \tag{6}$$

• Constraints on warehouse space

$$\sum_{n=1}^{N} v_{nt} I_{nt} \leq \widetilde{V}_{t\,max} \quad \forall t \tag{7}$$

• Constraint on total budget

$$z_1 \leq Z \tag{8}$$

• Non-negativity constraints on decision variables

$$Q_{nt}, O_{nt}, S_{nt}, I_{nt}, B_{nt}, H_t, F_t \geq 0 \quad \forall n, \forall t \tag{9}$$

3. Solution Methodology

3.1. Treatment of the fuzzy constraints

This work assumes the DM to have already adopted the triangular fuzzy number to represent the imprecise parameters in the fuzzy MOLP model formulated above. Recalling Eq. (4) from the original fuzzy MOLP model, consider the market demand, $\widetilde{\boldsymbol{D}}_{\boldsymbol{nt}}$, is a triangular fuzzy number with the most and least possible values. In the process of defuzzification, this work applies the weighted average technique to convert $\widetilde{\boldsymbol{D}}_{\boldsymbol{nt}}$ into a crisp number. If the minimum acceptable membership level, α, is given, the corresponding crisp expression of Constraints (4) can be presented as follows.

$$I_{nt-1} - B_{nt-1} + Q_{nt} + O_{nt} + S_{nt} - I_{nt} + B_{nt} = w_1 D^{p}_{nt,\ \alpha} + w_2 D^{m}_{nt,\ \alpha} + w_3 D^{o}_{nt,\ \alpha} \quad \forall n, \forall t \tag{10}$$

where, $\boldsymbol{w_1} + \boldsymbol{w_2} + \boldsymbol{w_3} = \boldsymbol{1}$, $\boldsymbol{w_1}$, $\boldsymbol{w_2}$ and $\boldsymbol{w_3}$ represent the weights of the most pessimistic, most likely and most optimistic values of the fuzzy market demand, respectively.

Similarly, the corresponding auxiliary crisp expressions of Constraints (6) and (7) can be presented as follows.

$$\sum_{n=1}^{N} l_{nt}(Q_{nt}+O_{nt}) = w_1 W^{p}_{t\,max,} + w_2 W^{m}_{t\,max,\ \alpha} + w_3 W^{o}_{t\,max,\ \alpha} \quad \forall t \tag{11}$$

$$\sum_{n=1}^{N} r_{nt}(Q_{nt}+O_{nt}) = w_1 M^{p}_{t\,max,} + w_2 M^{m}_{t\,max,\ \alpha} + w_3 M^{o}_{t\,max,\ \alpha} \quad \forall t \tag{12}$$

3.2. Strategy for solving the imprecise objective functions

The objective functions have triangular possibility distributions because the cost/time coefficients are always imprecise with triangular distributions. The strategy developed here for solving this imprecise objective function is to simultaneously minimize the most possible goal value, $\boldsymbol{z_1^m}$, maximize the possibility of obtaining lower goal value, $(\boldsymbol{z_1^m} - \boldsymbol{z_1^o})$, and minimize the risk of obtaining higher goal value, $(\boldsymbol{z_1^p} - \boldsymbol{z_1^m})$ [5]. The resulted three new objective functions of imprecise objective function (1) can be formulated as follows.

$$\begin{aligned} \text{Min } Z_{11} &= z_1^m \\ &= \sum_{n=1}^{N}\sum_{t=1}^{T}[a^m_{n1} Q_{nt}(1+i_a)^t + b^m_{n1} O_{nt}(1+i_b)^t + c^m_{n1} S_{nt}(1+i_c)^t \\ &+ d^m_{n1} I_{nt}(1+i_d)^t + e^m_{n1} B_{nt}(1+i_e)^t] \\ &+ \sum_{t=1}^{T}[k^m_t H_t(1+i_k)^t + m^m_t F_t(1+i_m)^t] \end{aligned} \tag{13}$$

$$\begin{aligned} \text{Max } Z_{12} &= z_1^m - z_1^p \\ &= \sum_{n=1}^{N}\sum_{t=1}^{T}\big[(a^m_{n1} - a^p_{n1}) Q_{nt}(1+i_a)^t + \big(b^m_{n1} - b^p_{n1}\big) O_{nt}(1+i_b)^t \\ &+ \big(c^m_{n1} - c^p_{n1}\big) S_{nt}(1+i_c)^t + (d^m_{n1} - d^p_{n1}) I_{nt}(1+i_d)^t + (e^m_{n1} - e^p_{n1}) B_{nt}(1 \\ &+ i_d)^t\big] + \sum_{t=1}^{T}\big[(k^m_t - k^p_t) H_t(1+i_k)^t + (m^m_t - m^p_t) F_1(1+i_m)^t\big] \end{aligned} \tag{14}$$

$$\text{Min } Z_{13} = z_1^o - z_1^m = \sum_{n=1}^{N}\sum_{t=1}^{T}[a_{n1}^o - a_{n1}^m)Q_{nt}(1+i_a)^t + (b_{n1}^o - b_{n1}^m)Q_{nt}(1+i_b)^t + (c_{n1}^o - c_{n1}^m)S_{nt}(1+i_c)^t + (d_{n1}^o - d_{n1}^m)I_{nt}(1+i_d)^t + (e_{n1}^o - e_{n1}^m)B_{nt}(1+i_e)^t] + \sum_{t=1}^{T}[(k_t^o - k_t^m)H_t(1+i_k)^t + (m_t^o - m_t^m)F_t(1+i_m)^t] \quad (15)$$

3.2. Fuzzy multi-objective linear programming model

3.3.1. Phase I: the minimum operator approach

In phase I, the original fuzzy MOLP problem can be solved using the fuzzy decision-making concept of Bellman and Zadeh [2] and Zimmermann [8]. The linear membership functions are specified for representing fuzzy goals involved, and the minimum operator is adopted to aggregate all fuzzy sets. By introducing the auxiliary variable $L^{(1)}$, the fuzzy MOLP problem can be converted into an equivalent ordinary single-goal LP model, as follows.

$$\text{Max } L^{(1)}$$

$$\text{s.t.} \quad L^{(1)} \le \frac{Z_{11}^{NIS} - Z_{11}}{Z_{11}^{NIS} - Z_{11}^{PIS}}$$

$$L^{(1)} \le \frac{Z_{12} - Z_{12}^{PIS}}{Z_{12}^{PIS} - Z_{12}^{NIS}}$$

$$L^{(1)} \le \frac{Z_{13}^{NIS} - Z_{13}}{Z_{13}^{NIS} - Z_{13}^{PIS}}$$

$$L^{(1)} \le \frac{Z_{2}^{NIS} - Z_{2}}{Z_{2}^{NIS} - Z_{2}^{PIS}}$$

$$0 \le L^{(1)} \le 1$$

Eqs. (4), (7), (8), (10)-(15) (16)

3.3.2. Phase II: the weighted average operator approach

In phase II, the initial solution obtained in Model (16) is improved by adding the lower bound of satisfaction degrees for each fuzzy objective function, $L^1{}_g (g=1,2,\ldots,K)$ as a constraint;

and the compensatory weighted average operator is then used to aggregate all fuzzy sets. By introducing the auxiliary variable $L^{(2)}$, the fuzzy MOLP problem can be converted into an ordinary LP model, as follows.

$$\text{Max } L^{(2)} = \lambda_{11}L_{11} + \lambda_{12}L_{12} + \lambda_{13}L_{13} + \lambda_2 L_2$$

$$\text{s.t.} \quad L_{11}^{l} \le L_{11} \le \frac{z_{11}^{NIS} - z_{11}}{z_{11}^{NIS} - z_{11}^{PIS}}$$

$$L_{12}^{l} \le L_{12} \le \frac{z_{12} - z_{12}^{PIS}}{z_{12}^{PIS} - z_{12}^{NIS}}$$

$$L_{13}^{l} \le L_{13} \le \frac{z_{13}^{NIS} - z_{13}}{z_{13}^{NIS} - z_{13}^{PIS}}$$

$$L_{2}^{l} \le L_{2} \le \frac{z_{2}^{NIS} - z_{2}}{z_{2}^{NIS} - z_{2}^{PIS}}$$

$$\lambda_{11} + \lambda_{12} + \lambda_{13} + \lambda_2 = 1$$

$$0 \le L^{(2)} \le 1$$

$$\text{Eqs. (4), (7), (8), (10)-(15)} \qquad (17)$$

4. Implementation

4.1. Data description

Daya Technologies Corporation was used as a case study to demonstrate the practicality of the proposed methodology. According to the preliminary production-marketing information, Tables 1 to 3 summarize the market demand, operating cost/time coefficients, related capacities, and warehouse space data used in the Daya case. Notably, the market demand, machine capacity and available labor levels are fuzzy numbers with triangular distributions. Other relevant data are as follows.

(1) Initial carrying inventory in period 1 is 400 units of product 1 and 200 units of product 2. The end inventory in period 4 is 300 units of product 1 and 200 units of product 2. The initial backordering volume in period 1 and the end backordering volume in period 4 for two products are zero.

(2) Initial labor level is 300 man-hours. The costs associated with hiring and layoffs are \$10 and \$2.5 per worker per hour, respectively.

(3) The expected escalating factor for each of the operating cost categories is fixed to 5% in each period. The minimum acceptable membership level is specified to 0.5 for each fuzzy parameter. The available total budget is $400,000.

Table 1. Fuzzy market demand data

Item	Period			
	1 (May)	2 (June)	3 (July)	4 (August)
$\widetilde{D}_{1t}$	(1,000, 900, 1,080)	(3,000, 2,750, 3,200)	(5,000, 4,600, 5,300)	(2,000, 1,850, 2,100)
$\widetilde{D}_{2t}$	(1,000, 900, 1080)	(500, 450, 540)	(3,000, 2,750, 3,200)	(2,500, 2,300, 2,650)

Table 2. Related unit cost/time coefficients data (in units of US dollars)

Product	$\tilde{a}_{nt}$ ($/unit)	$\tilde{b}_{nt}$ ($/unit)	$\tilde{c}_{nt}$ ($/unit)	$\tilde{d}_{nt}$ ($/unit)	$\tilde{e}_{nt}$ ($/unit)
1	(20, 17, 22)	(30, 26, 33)	(25, 22, 27)	(0.30, 0.27, 0.32)	(40, 35, 44)
2	(10, 8, 11)	(15, 12, 17)	(12, 10, 13)	(0.15, 0.13, 0.16)	(20, 16, 23)

Table 3. Maximum labor level, machine capacity and warehouse space data

Period	$\widetilde{W}_{t\max}$ (man-hours)	$\widetilde{M}_{t\max}$(machine-hours)	$V_{t\,max}$ (ft^2)
1	(300, 275, 320)	(400, 360, 430)	10,000
2	(300, 275, 320)	(500, 450, 540)	10,000
3	(300, 275, 320)	(600, 540, 650)	10,000
4	(300, 275, 320)	(500, 450, 540)	10,000

4.2. Solution procedure for the Daya case

In phase I, the fuzzy MOLP model for the APP decision is first formulated according to Eqs. (1) to (9). The equivalent ordinary LP model for solving the APP problem for the Daya case can be formulated according to Eqs. (16) and (17). LINGO software is used to run this ordinary crisp LP models. Table 4 lists initial and improved APP plans for the Daya case.

Table 4. Optimal APP plan for the Daya case

—	Initial solutions (phase I)	Improved solutions (phase II)
L	0.5595	0.7485
$\tilde{z}_1$ ($)	(354,764.3, 302,160.11, 386,938.45)	(335,144.7, 282,416.13, 367,663.72)
z_2 (man-hours)	562.75	325.39

4.3. Computational analysis and comparisons

The comparisons listed in Table 4 shows that the interaction of trade-offs and conflicts among dependent three objective functions. Analytical results obtained by implementing Daya case indicate that the proposed approach satisfies the requirement for the practical application since it attempts to simultaneously minimize the total product costs and total rates of changes in work-force level. Moreover, the proposed approach yields an efficient solution. The proposed two-phase fuzzy programming can overcome the disadvantage of using the minimum operator by adding phase I satisfaction degrees to phase II as a constraint, and the compensatory weighted average operator is employed for to obtain DM satisfaction degree. Finally, the fuzzy multi-objective APP model designed here considers the time value of money of related operating cost categories. The value of total production costs is impacted significantly by the monetary interest and, thus, a DM must consider the time value of money for each cost category when solving the real-life APP problems.

5. Conclusions

This work aims to develop a fuzzy mathematical programming approach for solving the fuzzy multi-objective APP problems with multiple products and multi-time periods. The main contribution of this work lies in presenting a fuzzy mathematical programming methodology to multi-objective APP decisions, and provides a systematic decision-making framework that facilitates a DM to interactively adjust the search direction until the preferred efficient solution is obtained.

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