Journal of Science and Engineering Technology, Vol. 4, No. 3, pp. 41-47 (2008)

A Tunable Damping and Natural-Frequency-Characteristic Analysis of an Orthotropic Annular Plate

JIA-YI YEH[1], JIUN-YEU CHEN[2], MING-YING HSU[3] and KUAN-CHUAN FANG[4]

[1]*Department of Management Information Science, Chung Hwa University of Medical Technology*

89, Wen-Hwa 1st St., Jen-Te Hsiang , Tainan Hsien 717

[2]*Department of Electronic Commerce, Hsing Kuo University of Management*

89, Yuying St., Tainan 709

[3]*Instrument Technology Research Center, National Applied Research Laboratories*

20 R&D Rd., VI, Hsinchu Science Park, Hsinchu 300

[4]*Tainan Branch, Bureau of Standards, Metrology and Inspection*

149, Beimen Rd., Sec. 2, Tainan 700

ABSTRACT

In this study, the equations of motion for a polar orthotropic annular sandwich plate are derived by using the discrete layer finite element method. The extensional and shear modulus of the electrorheological (ER) fluid layer are described by complex quantities; moreover, the tunable damping of the sandwich system is more effective when the electric fields are applied. The ER fluid core is found to have a significant effect on the vibrational behaviors of the plate. Finally, the effects of ER layer thickness, base annular plate stiffness, and certain designed parameters on natural frequencies and modal loss factors are also discussed.

Key Words: tunable damping, electrorheological (ER) layer, discrete layer finite element, polar orthotropic plate

極正交環板之可調式阻尼與自然振動頻率之特性分析

葉佳益[1]　陳俊宇[2]　徐名瑩[3]　方冠權[4]

[1]中華醫事科技大學資訊管理系

717 臺南縣仁德鄉文華一街 89 號

[2]興國管理學院電子商務學系

709 台南市安南區育英街 89 號

[3]國家實驗研究院儀器科技研究中心

300 新竹市科學園區研發六路 20 號

[4]經濟部標準檢驗局台南分局

700 台南市中區北門路 1 段 179 號

摘 要

本文主要在於推導並建立三明治極正交環板系統的運動方程式與分析模型的建立。採用分析力學（analytical mechanics）的方式來推導系統的運動方程式並利用離散層有限元素法（discrete layer finite element method）來分析系統的振動特性與阻尼變化。在本文中討論了含電變流體之三明治極正交環板系統的阻尼特性與自由振動，其中所使用的電變流體材料特性則利用複數的形式來加以描述。文中討論電變流體的厚度與施加的電場強度對於三明治極正交環板系統的特性影響，而系統的模態損失因子會隨著施加電場強度的改變而產生變化。因此，在此三明治極正交環板系統系統上貼覆此類主動式智能材料阻尼層除了可使系統更加的穩定外，也可達到主動控制的效果。

關鍵詞：可調式阻尼效應，電變流體，離散層有限元素，極正交

I. INTRODUCTION

The circular and annular plate had received the great deal of attention for the widely uses in many mechanical applications and the study on the circular and annular plate had been discussed by many researchers. Pandalai and Patel [6] studied the natural frequency of polar orthotropic circular plate for specific conditions. Then, Vijayakumar and Ramaiah [8] investigated the natural frequencies of the polar orthotropic circular and annular plates by using the Rayleigh-Ritz method. Lin and Tseng [4] studied the free vibration problems of polar orthotropic circular and annular plates. Recently, many investigations of the vibration and damping analysis for the mechanical structures can be found. Mirza and Singh [5] investigated the axisymmetric vibration of the sandwich circular plate. Roy and Ganesan [7] presented the finite element method to investigate the vibration and damping analysis of circular plate with constrained layer treatment. Yu and Huang [11] studied the problem of the three-layered circular and annular plate based on the thin shell theory to discuss the characteristics of the viscoelastic layer.

Recently, many studies on the active control of the structural vibration had been devoted to the use of the electrorheological (ER) material. The material properties of the ER material can vary with respect to applied electric field and the damping of the ER fluid had been paid much attention by many researchers since the Brooks *et al.* [1] studied the viscoelastic properties of the ER fluid. Choi and Park [2] carried out the investigation on the active vibration control and damping analysis of the cantilever sandwich beam with ER fluid. Then, Yalcintas and Coulter [9] adopted the ER material as controllable damping layer for the beam and plate configuration incorporating embedded sensors and control mechanism. The dynamic characteristics and damping effects of the sandwich isotropic and orthotropic rectangular plate structures were presented by Yeh and Chen [10].

In this paper, the vibration and damping behaviors of the sandwich polar orthotropic annular plate with ER core layer are studied. There are no works have been done to investigate this sandwich system with ER damping treatment to author's knowledge. The vibration and damping characteristics of the polar orthotropic annular plate with ER core layer are calculated by using the discrete layer finite element method. The extensional and shear modulus of the ER material are described by complex quantities and the natural frequencies and the modal loss factors of the sandwich system are obtained by solving the complex eigenvalue problem. Additionally, the effects of the ER layer and the influences of various parameters, such as thickness and applied electric fields are also discussed in this study.

II. PROBLEM FORMULATION OF THE SYSTEM

In Figure 1, the sandwich polar orthotropic annular plate with ER core layer is considered. Layer 1 is the constraining layer and assumed to be pure elastic and polar orthotropic. The ER core layer is designated as layer 2 and the material properties can be changed by applying various electric fields. Layer 3 is the base annular plate and assumed to be undamped, pure elastic, and polar orthotropic. The base annular plate is designed as the inner radius $r_i=a$ and outer radius $r_0=b$. Besides, the thickness for each layer is h_1, h_2, and h_3, respectively. In addition, the following assumptions must be mentioned first. The transverse displacements of each layer are equal and there are no slipping between the constraining-ER layer and ER-plate layer.

In order to analyze the problem, the displacement field of the layer i is employed as follows:

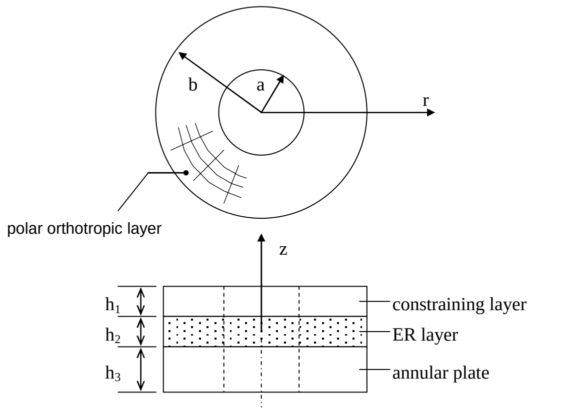

Fig. 1. Polar orthotropic sandwich annular plate system

$$d_i = \begin{Bmatrix} u_i(r,\theta,z,t) \\ w_i(r,\theta,z,t) \end{Bmatrix} = \begin{bmatrix} (\frac{1}{2}-\frac{z}{h_i}) & (\frac{1}{2}+\frac{z}{h_i}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} U_i(r,\theta,t) \\ U_{i+1}(r,\theta,t) \\ W(r,\theta,t) \end{Bmatrix}$$

$$= H_1(z) \begin{Bmatrix} U_i(r,\theta,t) \\ U_{i+1}(r,\theta,t) \\ W(r,\theta,t) \end{Bmatrix}, \qquad (1)$$

By using the interpolation in r-direction and the circumferential wave number *m*, the displacements of the interfaces for two-layer can be shown in terms of the nodal degrees of freedom as follows:

$$\begin{Bmatrix} U_i(r,\theta,t) \\ U_{i+1}(r,\theta,t) \\ W(r,\theta,t) \end{Bmatrix} = \begin{bmatrix} \phi_u^A & 0 & 0 & 0 & \phi_u^B & 0 & 0 & 0 \\ 0 & \phi_u^A & 0 & 0 & 0 & \phi_u^B & 0 & 0 \\ 0 & 0 & \phi_w^A & \phi_\Theta^A & 0 & 0 & \phi_w^B & \phi_\Theta^B \end{bmatrix} q_i^e(t)$$

$$= H_2(r) q_i^e(t), \qquad (2)$$

where the vector of the nodal displacements of the *i*th layer element, as shown in Figure 2,

$$q_i^e(t) = \begin{Bmatrix} U_i^A & U_{i+1}^A & W^A & \Theta^A & U_i^B & U_{i+1}^B & W^B & \Theta^B \end{Bmatrix}^T,$$

and $H_2(r)$ is the interpolation matrix and in which

$$\phi_u^A = (1-\xi)\cos m\theta, \ \phi_u^B = \xi \cos m\theta,$$

$$\phi_w^A = (1 - 3\xi^2 + 2\xi^3)\cos m\theta,$$

$$\phi_w^B = (3\xi^2 - 2\xi^3)\cos m\theta,$$

$$\phi_\Theta^A = (\xi - 2\xi^2 + \xi^3)\cos m\theta,$$

$$\phi_\Theta^B = (-\xi^2 + \xi^3)\cos m\theta,$$

$$\xi = (r - r_i)/(r_0 - r_i).$$

Then, the strain-displacement relation for the *i*th layer of the system can be expressed as the following form:

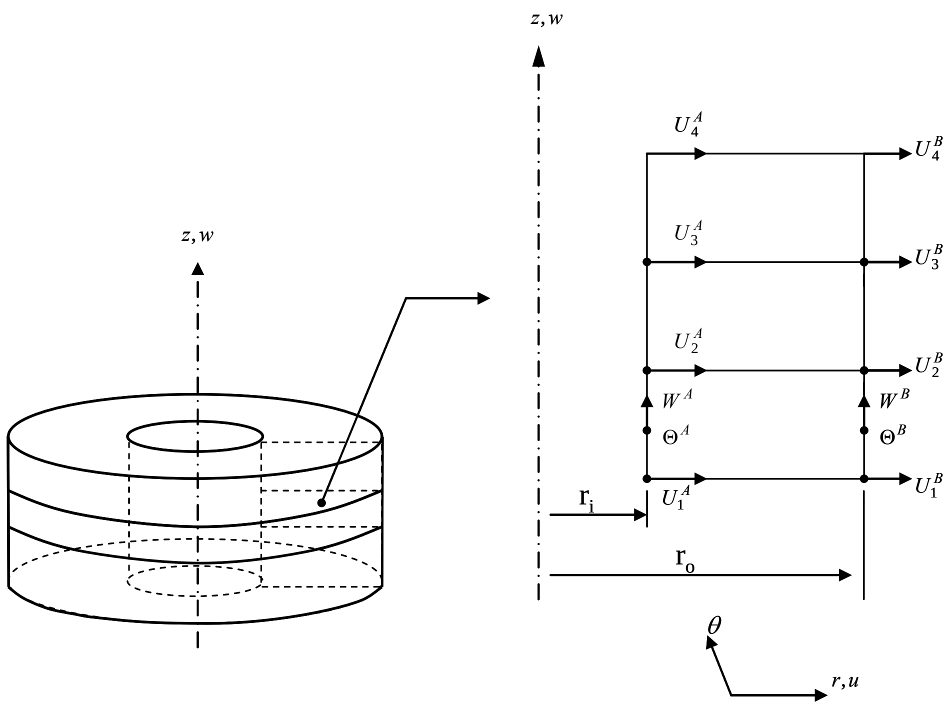

Fig. 2. Three-layer discrete layer annular finite element

$$\varepsilon_i = \begin{Bmatrix} \varepsilon_{r,i} \\ \varepsilon_{\theta,i} \\ \gamma_{rz,i} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial r} & 0 \\ \frac{1}{r} & 0 \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial r} \end{bmatrix} d_i \,, \tag{3}$$

Then, the stress-strain relation can be obtained and shown as follows:

$$\sigma_i = \begin{Bmatrix} \sigma_{r,i} \\ \sigma_{\theta,i} \\ \tau_{r\theta,i} \end{Bmatrix} = \begin{bmatrix} C_{11,i} & C_{12,i} & 0 \\ C_{21,i} & C_{22,i} & 0 \\ 0 & 0 & C_{44,i} \end{bmatrix} \varepsilon_i \,, \tag{4}$$

In which, for isotropic ER material,

$$C_{11,2} = C_{22,2} = \frac{E_2}{1-\upsilon_2^2} \,,$$

$$C_{12,2} = C_{21,2} = \frac{\upsilon_2 E_2}{1-\upsilon_2^2} \,,$$

$$C_{44,2} = \frac{E_2}{2(1+\upsilon_2)} \,,$$

$$\upsilon_2 = 0.499 \,,$$

respectively. For polar orthotropic material,

$$C_{11,i} = \frac{E_{r,i}}{1-\upsilon_{r\theta,i}\upsilon_{\theta r,i}} \,,$$

$$C_{22,i} = \frac{E_{\theta,i}}{1-\upsilon_{r\theta,i}\upsilon_{\theta r,i}} \,,$$

$$C_{12,i} = C_{21,i} = \frac{\upsilon_{\theta r,i} E_{r,i}}{1-\upsilon_{r\theta,i}\upsilon_{\theta r,i}} \,,$$

$$C_{44,i} = \kappa G_{rz,i} \,,$$

respectively. In the above equations, E_i is the Young's modulus, υ_i is the Poisson ration, and κ is the shear correction factor.

According to the above equation, the kinetic and strain energies of the element for *i*th layer can be expressed as the following form:

$$T_i^e = \frac{1}{2}\int_V \rho_i \dot{d}_i^{\mathrm{T}} \dot{d}_i dV \,, \quad V_i^e = \frac{1}{2}\int_V \rho_i \sigma_i^{\mathrm{T}} \varepsilon_i dV \,, \tag{5}$$

where ρ_i is the mass density of the *i*th layer.

The kinetic and strain energies of the element can be rewritten as follows by substituting the equation (1), (2), (3), and (4):

$$V_i^e = \frac{1}{2} \mathrm{U}_i^{e\mathrm{T}} \mathrm{K}_i^e \mathrm{U}_i^e \,, \quad T_i^e = \frac{1}{2} \dot{\mathrm{U}}_i^{e\mathrm{T}} \mathrm{M}_i^e \dot{\mathrm{U}}_i^e \,, \tag{6}$$

Then, the following relations must be obtained by combining the elemental matrices into the global stiffness and mass matrices:

$$U_i^e = Tr_i^e U \,, \tag{7}$$

where U and Tr_i^e are the global nodal co-ordinate vector and transformation matrix, respectively.

The equation of motion for the polar orthotropic annular sandwich system can be express as the following form by assembling the contributions of all elements:

$$M\ddot{U} + KU = 0 \,, \tag{8}$$

in which,

$$K = \sum_{i=1}^{3}\left(\sum_{e=1}^{N_i} Tr_i^{e\mathrm{T}} K_i^e Tr_i^e\right), \quad M = \sum_{i=1}^{3}\left(\sum_{e=1}^{N_i} Tr_i^{e\mathrm{T}} M_i^e Tr_i^e\right), \tag{9}$$

where N_i is the element number of the *i*th layer.

Finally, the complex eigenvalues $\tilde{\lambda}$ of the above complex eigenvalue problems can be calculated numerically. The natural frequencies ω and modal loss factor η_v of the sandwich polar orthotropic annular plate with ER core layer can be obtained as follows:

$$\omega = \sqrt{Re(\tilde{\lambda})} \,, \quad \eta_v = \frac{Im(\tilde{\lambda})}{Re(\tilde{\lambda})} \,. \tag{10}$$

III. RESULTS AND DISCUSSIONS

In this paper, the discrete layer finite element method is adopted to calculate the vibration problem of the sandwich system with ER core treatment. The damping effects of the sandwich system are provided by the ER fluid in this study, and only the electric field dependence of ER fluid needed to consider based on the existing model of ER material. The calculations of the natural frequencies and modal loss factors for the sandwich annular plate are also presented in Table 1, and the boundary conditions are clamped at inner edge and free at outer edge and the number of elements in the r-direction is taken to be 16. Good accuracy and convergence can be found in the above comparisons. The complex modulus of ER fluid can be simplified into the following form, which was experimentally calculated by Don [3]:

Table 1. Comparisons between published and proposed methods for the full coverage annular plate

Mode	Natural frequency (Hz)		Modal loss factor	
(n, m)	Present	Ref. [7]	Present	Ref. [7]
(0,0)	74.44	74.38	0.11280	0.11270
(0,1)	73.00	73.08	0.09542	0.09576
(0,2)	96.20	96.38	0.10160	0.10210
(0,3)	144.00	142.80	0.12100	0.12120
(0,4)	205.20	203.70	0.11700	0.11770

$$G_2(E_*) = G' + jG'' , \tag{11}$$

in which, $j = \sqrt{-1}$, $G' \approx 15000E_*^2$, $G'' \approx 6900$ and E_* is the applied electric field in *kV/mm* .

Additionally, in order to simplify the following analysis and discussion, the geometric and non-dimensional parameters are used:

$$\tilde{a} = \frac{a}{b} ,\ \tilde{h}_2 = h_2/h_3 ,\ \tilde{h}_1 = h_1/h_3 ,\ \tilde{E}_1 = \frac{E_{\theta,1}}{E_{r,1}} ,\ \tilde{E}_3 = \frac{E_{\theta,3}}{E_{r,3}} ,$$

$\upsilon_{\theta r,i}$= 0.29, υ_2 = 0.49, $\rho_1 = \rho_3$ = 2700 kg/m^3,

ρ_2 = 1700 kg/m^3, b=0.15 m, $E_{r,1}$= $E_{r,3}$=70 *GPa*,

$\kappa = \pi^2/12$ (for layer 1, 3), $\kappa = 1$ $\kappa = 1$ (for layer 2).

The effects of applied electric fields on the natural frequency and modal loss factor of the polar orthotropic sandwich annular plate for mode (0, 0), (0, 1) and (0, 2) are shown in Figure 3. Additionally, the parameters of the system are $\tilde{E}_1 = 1$, $\tilde{E}_3 = 1.5$, $\tilde{a} = 0.1$, b=0.15 m, h_3=0.5 *mm*, $\tilde{h}_{13} = 0.1$, $\tilde{h}_{23} = 0.5$, respectively. As shown in the figure, it can be observed that the natural frequency increases as the applied electric field magnitude increases. Besides, it also can be found that the modal loss factor of the system decreases as the applied electric field increases. The tendency for each mode is the same from above numerical results.

Figure 4 presents the effects of $\tilde{E}_3$ on the natural frequency and modal loss factor of the sandwich system with various thickness of ER layer for mode (0, 0). The parameters of the system are $\tilde{E}_1 = 1$, $\tilde{a} = 0.1$, E_* = 0.5 *kV/mm*, b = 0.15 m, h_3=0.5 *mm*, $\tilde{h}_{13} = 0.1$, respectively. According to the above results, it can be seen that the natural frequency becomes larger when $\tilde{E}_3$ increases. In addition, the modal loss factor will decrease as $\tilde{E}_3$ increases. On the other hand, the natural frequency decreases and modal loss factor increase when the thickness of ER layer increases.

In Figure 5, the numerical results of the effects of $\tilde{E}_3$ on the natural frequency and modal loss factor of the sandwich system with various electric fields for mode (0, 0) can be

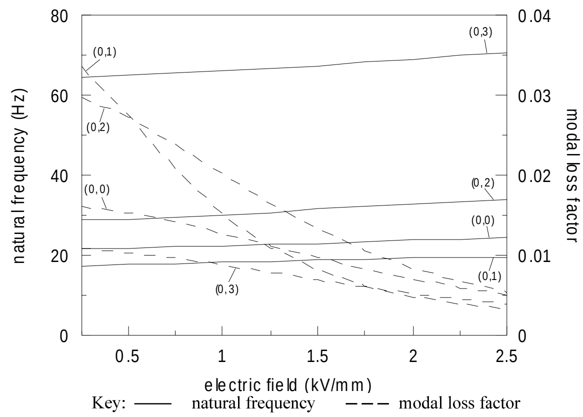

Fig. 3. Effects of electric fields on the natural frequencies and the modal loss factors of the sandwich annular plate

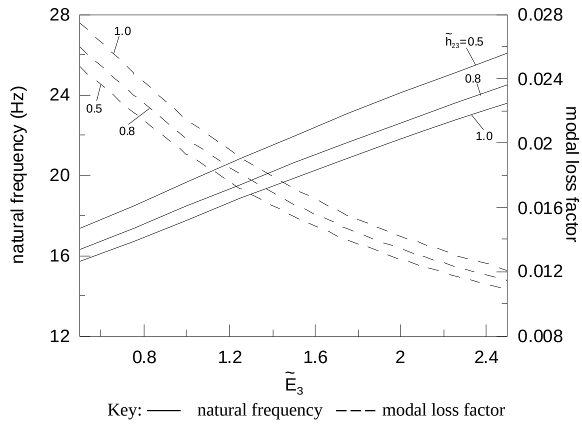

Fig. 4. Effects of $\tilde{E}_3$ on the natural frequencies and the modal loss factors of the sandwich annular plate with various thickness of ER layer

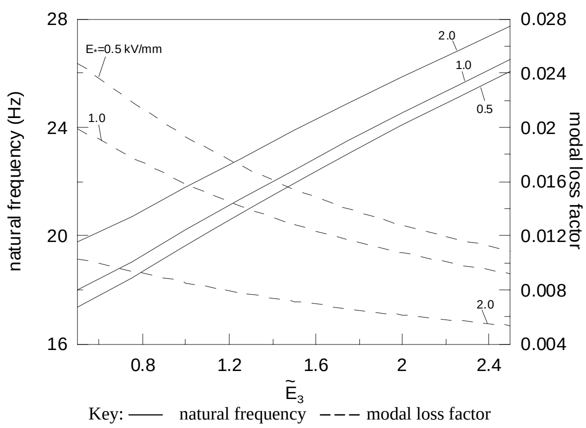

Fig. 5. Effects of $\tilde{E}_3$ on the natural frequencies and the modal loss factors of the sandwich annular plate with various applied electric fields

obtained. The parameters of the system are $\tilde{E}_1 = 1$, $\tilde{a} = 0.1$, b = 0.15 *m*, h_3 = 0.5 *mm*, $\tilde{h}_{13} = 0.1$, $\tilde{h}_{23} = 0.5$. From the figure, it can be observed that the natural frequency increase as the applied electric fields increases. On the contrary, the modal loss factor decreases while the applied electric field increases. Therefore, the damping characteristics of the sandwich system can be controlled by the applied electric fields.

The effects of thickness of ER layer on the natural frequency and modal loss factor with various ratios $\tilde{E}_3$ are presented in Figure 6. The parameters of the system are $\tilde{E}_1 = 1$, $\tilde{a} = 0.1$, $E*$ = 0.5 *kV/mm*, b = 0.15 *m*, h_3 = 0.5 *mm*, $\tilde{h}_{13} = 0.1$, respectively. According to the numerical results, it can be observed that the larger thickness of the ER layer, the smaller natural frequency. Contrary to the natural frequency, the larger thickness of ER layer, the larger modal loss factor. Besides, the natural frequency will increase and modal loss factor will decrease while $\tilde{E}_3$ increases from the figure.

IV. CONCLUSIONS

The damping characteristics problems of the polar orthotropic sandwich annular plate with ER core layer are investigated in this study. The discrete layer finite element method is adopted to calculate the sandwich orthotropic annular system. Besides, the complex description of viscoelastic material is used for the ER fluid. From the numerical results, the following conclusions can be presented. It can be observed that the ER damping treatment can make the system stable from the numerical results. And, the results show that the natural frequency and modal loss factor will vary with changing the strength ratio $\tilde{E}_3$.

The applied electric field can change the vibration and damping characteristics of the polar orthotropic sandwich system. The thickness of the ER layer also can change the stiffness of the sandwich system, and the natural frequency and modal loss factor of the system will be change. Besides, the damping effects of the ER layer can be changed by applying different electric fields and shown to have significant variations on the vibration and damping characteristics.

According to the present numerical results, the present designs can be used in practical mechanical applications to achieve active controllable system. Besides, it can be used to design the smart devices and other mechanical applications as basic information for the future practical applications.

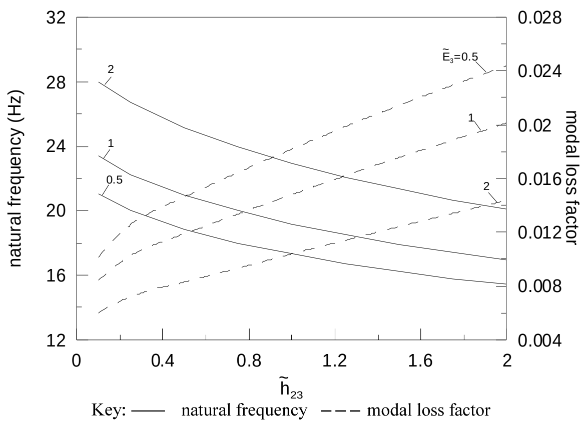

Fig. 6. Effects of thickness of ER layer on the natural frequencies and the modal loss factors of the sandwich annular plate with various ratios $\tilde{E}_3$

ACKNOWLEDGEMENT

This research was partially supported by the National Science Council in Taiwan through Grant NSC 96-2221-E-273-009-MY3.

REFERENCES

1. Brooks, D. A., J. Goodwin, C. Hjelm, L. Marshall and C. Zukoski (1986) Viscoelastic studies on an electro-rheological fluid. *Colloids & Surfaces*, 18, 293-312.
2. Choi, S. B. and Y. K. Park (1994) Active vibration control of cantilever beam containing an electro-rheological fluid. *Journal of Sound Vibration*, 172, 428-432.
3. Don, D. L. (1993) *An Investigation of Electrorheological Material Adoptive Structures*, Master's Thesis Lehigh University, Bethlehem, Pennsylvania.
4. Lin, C. C. and C. S. Tseng (1998) Free vibration of polar orthotropic laminated circular and annular plates. *Journal of Sound Vibration*, 209, 797-810.
5. Mirza, S. and A. V. Singh (1974) Axisymmetric vibration of circular sandwich plates. *AIAA Journal*, 12, 1418-1420.
6. Pandalai, K. A. V. and S. A. Patel (1965) Natural frequencies of orthotropic circular plates. *AIAA Journal*, 3, 780-781.
7. Roy, P. K. and N. Ganesan (1993) A vibration and damping analysis of circular plates with constrained damping layer treatment. *Computers and Structures*, 49, 269-274.
8. Vijayakumar, K. and G. K. Ramaiah (1974) Estimation of higher natural frequencies of polar orthotropic annular plates. *Journal of Sound Vibration*, 32, 265-278.
9. Yalcintas, M. and J. P. Coulter (1995) Analytical modeling of electrorheological material based adaptive beams. *Journal of Intelligent Material Systems and Structures*, 6, 488-497.

10. Yeh, J. Y. and L. W. Chen (2005) Dynamic stability of a sandwich plate with a constraining layer and electrorheological fluid core. *Journal of Sound Vibration*, 285, 637-652.
11. Yu, S. C. and S. C. Huang (2001) Vibration of a three-layered viscoelastic sandwich circular plate. *International Journal of Mechanical Sciences*, 43, 2215-2236.

Received: Feb. 27, 2008 Revised: Mar. 28, 2008
Accepted: May 1, 2008