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Dynamic Design of Beams Using Soft Tuning

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ABSTRACT

A technique for shifting the natural frequencies of beam structures to designated values by using a soft-tuning method is proposed. When the stiffness of beams is changed, the natural frequencies are also altered. Structural stiffness can be increased by enhancing the structures, e.g., strengthening them. Similarly, the stiffness can also be reduced by softening the structures. The natural frequencies can be shifted to the desired values by tuning the depths and locations of cracks, the procedure which is proposed in this report. This method utilizes a transfer matrix technique to obtain an analytical characteristic equation of the system, on which the dynamic frequency tuning is based. Two examples using this method are demonstrated and experimentally validated.
***Key Words*:** soft tuning, natural frequency, transfer matrix, characteristic equation

以軟調法做樑之動態設計

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摘　要

本文主要提出一方法使樑結構之自然頻率調整至一指定頻率值。通常結構勁度改變，其自然頻率亦隨之而變。增加結構勁度能使結構加強；相同的，使結構減弱亦可調整其自然頻率。本文使用人為加一裂縫至樑結構，使樑結構之自然頻率調至一指定之頻率值。先使用轉移矩陣法得系統之特徵方程式，再利用此特徵方程式調整此系統之動態特性。本文並附實例說明及實驗驗證。
關鍵詞：軟調整，自然頻率，轉移矩陣，特徵方程式

I. INTRODUCTION

Shifting or moving the structural resonance out of a specific frequency range, is advantageous to reducing the structural vibroacoustic response [13]. Most of the previous studies related to shifting the natural frequencies away from the forcing frequency or to a higher value for structures are based on optimization methods. Different optimization strategies related to the minimization of the vibroacoustic response have been developed and can be found in the literature [5].

For most of the cases, structural stiffness is increased to make the structures stronger by changing the dimensions of the structural components or by including some extra structural components. For some structural systems, the natural frequencies are altered through the addition of auxiliary components such as masses, ribs, *etc*. The addition of an auxiliary "spring-mass-damper" system is an effective strategy to control the natural frequencies of the system [4, 9]. Generally, these studies focused on determining the natural frequencies associated with different boundary conditions and were limited to straightforward vibration response calculations. Wang and Cheng [13] used the structural patches to change the structural natural frequencies. Some studies used structure dimples to modify the vibration characteristics [1].

In this paper, the natural frequencies of beam structures are changed to designated values by using soft tuning methods, that is, beam stiffness is reduced by making beam structures softer. The easiest way to make beams softer is to create cracks on beams artificially. The existence of cracks will alter the dynamic characteristics of structures. The dynamics of cracked structures has been a topic of active research during the last decade. When a structural component is subjected to a crack, the crack induces a local flexibility which is a function of the crack depth, thereby changing its dynamic behavior and its stability characteristics. The dynamic behavior of the cracked structures was studied by several analytical and numerical methods [8, 10, 11]. Dimarogonas [3] presents a state of review on the dynamics of the cracked structures. A complete cracked beam vibration theory is also developed by Chondros and Dimarogonas [2] for the lateral vibration of cracked Euler-Bernoulli beam with single-edge or double-edge open cracks. In [2], the crack region as a local flexibility was expressed by a crack disturbance function $f(x, z)$ which could be derived from the stress intensity factors in the theory of fracture mechanics. In most of the previous studies, the model of Euler-Bernoulli beam theory by deriving the differential equation and the associated boundary conditions for a uniform Euler-Bernoulli beam containing one or two cracks are often used and discussed. Lin et al. [8] presents a hybrid analytical/numerical method that permits the efficient computation of the eigensolutions for an arbitrary number of cracks of a beam with various boundary conditions. The method is based on the use of massless rotational spring to present the cracks, and by the compatibility conditions of each crack, the relationships of the four integration constants of the eigenfunctions between adjacent sub-beams can be determined [8]. By using the transfer matrix, the characteristic equation of the system can be obtained analytically. The depths and locations of cracks control the dynamic characteristics of this system. In this way, the dynamics of beams can be shifted to specific values through the method of crack tuning.

II. THEORETICAL MODEL

An Euler-Bernoulli beam of length L and with $k$open cracks is considered as in Fig. 1. It is assumed that the cracks are located at points X_1, X_2,…, X_k, such that $0< X_1 < X_2 <\ldots< X_k < L$. The vibration amplitudes of the transverse displacements of the beam are denoted by $Y_i(X, T)$ on the interval $X_{i-1}< X <X_i$, where the sub-index i represents the i-th segment and i=1, 2,…, k+1 (refer to Fig. 1). The entire beam (whole domain) is now divided into (k+1) segments with lengths L_1, L_2,…, L_{k+1}, respectively which are separated by k cracks. The equation of motion for each segment, assuming with uniform cross section, is [8]:

$$EI\frac{\partial^4 Y_i(X,T)}{\partial X^4}+\rho A\frac{\partial^2 Y_i(X,T)}{\partial T^2}=0\,,$$

$$X_{i-1}<X<X_i,\qquad i=1,2,\ldots,k+1 \tag{1}$$

where E is Young's modulus of the material, I is the moment of inertia of the beam cross-section, ρ is the density of material and A is the cross-section area of the beam.

The boundary conditions of the beam for the simply supported case are:

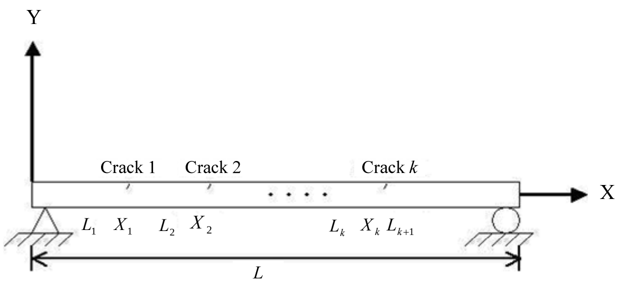

Fig. 1. A beam with k cracks located at positions X_1, X_2,…, X_k, respectively, and the sub-domains are L_1, L_2,…, L_k, L_{k+1}, where $L_1+L_2+\ldots+L_k+L_{k+1}=L$.

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$Y(0, T) = Y(L, T)=0,$ (2, 3)

$Y''(0, T) = Y''(L, T)=0,$ (4, 5)

The "compatibility conditions" enforce continuities of the displacement field, bending moment and shear force respectively across each crack and can be expressed:

$$Y_i(X_i^-, T) \;=\; Y_{i+1}(X_i^+, T)\,, \tag{6}$$

$$Y_i''(X_i^-, T) \;=\; Y_{i+1}''(X_i^+, T)\,, \tag{7}$$

$$Y_i'''(X_i^-, T) \;=\; Y_{i+1}'''(X_i^+, T)\,,\; i=1, 2,\ldots, k \tag{8}$$

where the symbols X_i^+ and X_i^- denote the locations immediately above and below the cracks. Moreover, a discontinuity into the slope of the beam across each crack exists and can be expressed [8]:

$$Y_{i+1}'(X_i^+, T) - Y_i'(X_i^-, T) \;=\; \theta_i L Y_{i+1}''(X_i^+, T)\,,\; i=1, 2,\ldots, k \tag{9}$$

where θ_i is the non-dimensional i-*th* crack section flexibility which are functions of the crack extent [11]. For double-side open cracks, refer to Fig. 2(a) [11]:

$$\theta_i = 6\pi\bar{\gamma}_i^{\,2}\, f_D(\bar{\gamma}_i)\,(\frac{H}{L}) \tag{10}$$

where $\bar{\gamma}_i = \dfrac{a_i}{H}$, a_i is the depth of the i-*th* crack and

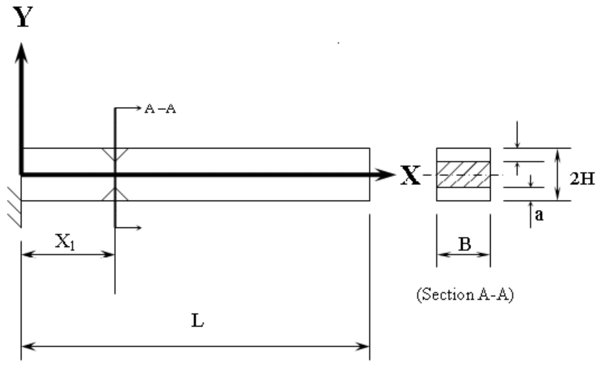
$$f_D(\bar{\gamma}_i) = 0.5335 - 0.929\bar{\gamma}_i + 3.500\bar{\gamma}_i^{\,2} - 3.181\bar{\gamma}_i^{\,3} + 5.793\bar{\gamma}_i^{\,4}\,. \tag{11}$$

For single-side open cracks, refer to Fig. 2(b) [11]:

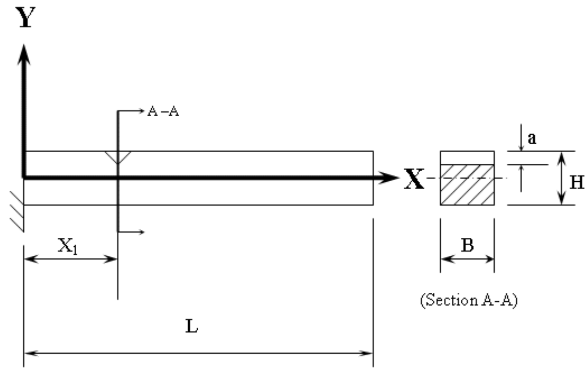
$$\theta_i = 6\pi\bar{\gamma}_i^{\,2}\, f_J(\bar{\gamma}_i)\,(\frac{H}{L}) \tag{12}$$

$$f_J(\bar{\gamma}_i) = 0.6384 - 1.035\bar{\gamma}_i + 3.7201\bar{\gamma}_i^{\,2} - 5.1773\bar{\gamma}_i^{\,3} + 7.553\bar{\gamma}_i^{\,4} - 7.332\bar{\gamma}_i^{\,5} + 2.4909\bar{\gamma}_i^{\,6}\,. \tag{13}$$

In the above, the following transformation quantities are introduced:



(a) Sketch of a double-side open crack



(b) Sketch of a single-side open crack

Fig. 2. Double-side and single-side open cracks

$$y \;=\; \frac{Y}{L}\,, \tag{14}$$

$$x \;=\; \frac{X}{L}\,, \tag{15}$$

$$t \;=\; \frac{T}{\sqrt{L}}\,, \tag{16}$$

$$l_i = \frac{L_i}{L}\,, \tag{17}$$

$$x_i = \frac{X_i}{L}\,. \tag{18}$$

Thus, in each segment, Eq. (1) can then be expressed in the form as:

$$\frac{EI}{L^3}\frac{\partial^4 y_i(x,t)}{\partial x^4} \;+\; \rho A\,\frac{\partial^2 y_i(x,t)}{\partial t^2} \;=\; 0\,,$$

$$x_{i-1} < x < x_i\,, \qquad i = 1,2,\ldots,k+1\,. \tag{19}$$

The transformations of “compatibility conditions” from Eqs. (6) to (9) are:

$$y_i(x_i^-,t) = y_{i+1}(x_i^+,t)\,, \tag{20}$$

$$y_i''(x_i^-,t) = y_{i+1}''(x_i^+,t)\,, \tag{21}$$

$$y_i'''(x_i^-,t) = y_{i+1}'''(x_i^+,t)\,, \tag{22}$$

$$y_{i+1}'(x_i^+,t) - y_i'(x_i^-,t) = \theta_i y_{i+1}''(x_i^+,t)\,, \tag{23}$$

where i=1, 2,…, k and θ_i is in Eqs. (10) and (12) for double-side and single-side open cracks respectively.

III. SOFT TUNING BY CRACKS

The eigensolutions for the cases of commonly used different boundary conditions are derived by the transfer matrix method [6, 7]. For the examples of one crack case, the characteristic equations of the system for different boundary conditions are obtained and are re-written here as [6]:

1. Simply-Supported Beam

$$\begin{aligned}&4\sin\lambda_n \sinh\lambda_n + \theta_1\lambda_n(\cos\lambda_n \sinh\lambda_n + \sin\lambda_n \cosh\lambda_n)\\ &-\theta_1\lambda_n[\cos(\lambda_n(1-2l_1))\ \sinh\lambda_n + \sin\lambda_n\ \cosh(\lambda_n(1-2l_1))] = 0\end{aligned} \tag{24}$$

2. Cantilever Beam

$$\begin{aligned}&4+4\cos\lambda_n \cosh\lambda_n + \theta_1\lambda_n(\cos\lambda_n \sinh\lambda_n - \sin\lambda_n \cosh\lambda_n)\\ &+\theta_1\lambda_n[\cos(\lambda_n(1-2l_1))\sinh\lambda_n - \sin\lambda_n\ \cosh(\lambda_n(1-2l_1))]\\ &+2\,\theta_1\lambda_n[\cos(\lambda_n(1-l_1))\ \sinh(\lambda_n(1-l_1))\\ &\qquad -\sin(\lambda_n(1-l_1))\cosh(\lambda_n(1-l_1))]\\ &+2\,\theta_1\lambda_n[\sin(\lambda_n l_1)\ \cosh(\lambda_n l_1) - \cos(\lambda_n l_1)\ \sinh(\lambda_n l_1)] = 0\end{aligned} \tag{25}$$

3. Fixed-Fixed Beam

$$\begin{aligned}&-4+4\cos\lambda_n \cosh\lambda_n + \theta_1\lambda_n(\cos\lambda_n \sinh\lambda_n - \sin\lambda_n \cosh\lambda_n)\\ &+\theta_1\lambda_n[\cos(\lambda_n(1-2l_1))\sinh\lambda_n - \sin\lambda_n\ \cosh(\lambda_n(1-2l_1))]\\ &+2\,\theta_1\lambda_n[\cos(\lambda_n(1-l_1))\ \sinh(\lambda_n(1-l_1))\\ &\qquad -\sin(\lambda_n(1-l_1))\cosh(\lambda_n(1-l_1))]\\ &-2\,\theta_1\lambda_n[\sin(\lambda_n l_1)\ \cosh(\lambda_n l_1) - \cos(\lambda_n l_1)\ \sinh(\lambda_n l_1)] = 0\end{aligned} \tag{26}$$

4. Free-Free Beam

$$\begin{aligned}&4-4\cos\lambda_n \cosh\lambda_n - \theta_1\lambda_n(\cos\lambda_n \sinh\lambda_n - \sin\lambda_n \cosh\lambda_n)\\ &-\theta_1\lambda_n[\cos(\lambda_n(1-2l_1))\sinh\lambda_n - \sin\lambda_n\ \cosh(\lambda_n(1-2l_1))]\\ &+2\,\theta_1\lambda_n[\cos(\lambda_n(1-l_1))\ \sinh(\lambda_n(1-l_1))\\ &\qquad -\sin(\lambda_n(1-l_1))\cosh(\lambda_n(1-l_1))]\\ &-2\,\theta_1\lambda_n[\sin(\lambda_n l_1)\ \cosh(\lambda_n l_1) - \cos(\lambda_n l_1)\ \sinh(\lambda_n l_1)] = 0\end{aligned} \tag{27}$$

where θ_1 is the non-dimensional crack section flexibility in Eq. (10) and (12) for double and single-side open cracks respectively, l_1 is the non-dimensional crack location, and λ_n are the eigenvalues corresponding to the natural frequencies ω_n of the system as [6]:

$$\lambda_n^4 = \frac{\rho A\omega_n^2 L^3}{EI}\,. \tag{28}$$

In the above equations, there are three parameters: (1) θ_1, (2) l_1 and (3) λ_n. If some specific designated natural frequency of the beam ω_n is required, substituting the λ_n value corresponding to this natural frequency from Eq. (28) to these equations (Eq. (24) ~ Eq. (27)), the required relationship of θ_1 and l_1 can then be obtained.

IV. NUMERICAL RESULTS AND DISCUSSION

In order to show the procedure of soft tuning by cracks and validate the method presented in this article, two numerical examples are used and compared with the available data.

***Example A*:** For the case of a cantilever beam, as ref. [12], the beam parameters are: beam length L=300 *mm*, width B=20.0 *mm*, height H=20.0 *mm*, Young’s modulus E=2.06×10^{11} N/m^2, density ρ=7,800 Kg/m^3. First, a one-single-side open crack is used to tune the natural frequency of this beam to a designated value of 174.0 Hz. The corresponding eigenvalue to this frequency is λ_1=1.82103 ($\lambda_1^4 = \frac{\rho A\,\omega_1^2 L^3}{EI}$). Substituting this value into Eq. (25) and the relationship between the non-dimensional crack location (l_1) and the non-dimensional crack flexibility (θ_1) can be obtained as shown in Fig. 3.

With any pair of l_1 and θ_1 values on this curve, we can obtain the designated beam natural frequency of 174.0 Hz. For example, if the crack location is chosen to be l_1 =0.55, then the

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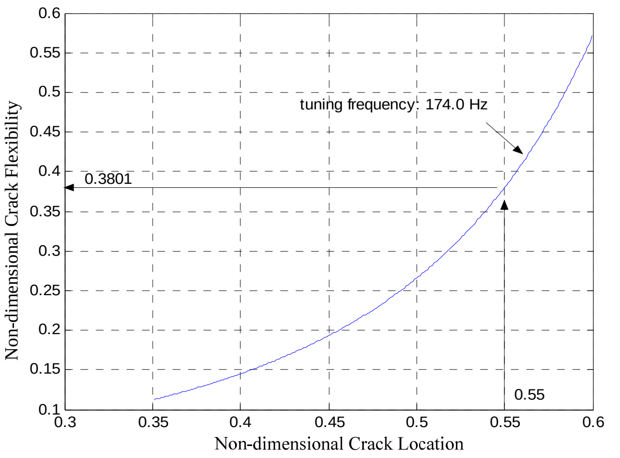

Fig. 3. The relationship between the non-dimensional crack location l_1 and the non-dimensional crack flexibility θ_1 using one tuning frequency.

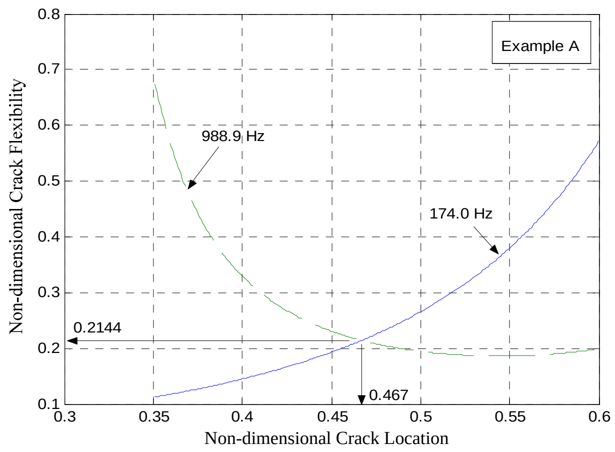

Fig. 5. The determination of the non-dimensional crack location l_1 and non-dimensional crack flexibility θ_1 using two tuning frequencies for *Example A*.

corresponding crack flexibility is θ_1=0.3801 (refer to Fig. 3). When the value of crack flexibility θ_1 is determined, the non-dimensional crack depth $\bar{\gamma}_1$ can then be obtained from Eq. (12) and the value is 0.623 in this case as shown in Fig. 4.

The above procedure is used to tune the beam to one designated natural frequency. Moreover, two desired natural frequencies can also be tuned simultaneously by using this method. For the above example, if two designated natural frequencies, i.e., 174.0 Hz and 988.9 Hz, are required, using the same procedure, two curves can be obtained as shown in Fig. 5. The intersection of curves in Fig. 5 is the point required to satisfy these two frequencies. The solution of these two curves is l_1 =0.467 and θ_1=0.2144 (refer to Fig. 5).

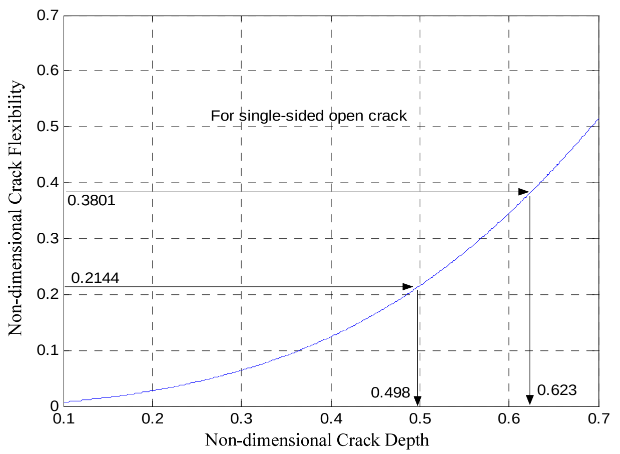

Fig. 4. The relationship between the non-dimensional crack depth $\bar{\gamma}_1$ and the non-dimensional crack flexibility θ_1.

The non-dimensional crack depth corresponding to this value of crack flexibility is $\bar{\gamma}_1 = 0.498$, referring the curve in Fig. 4. In the experiment in Rizos and Aspragathos [12], the crack is located 140 *mm* from the fixed end (l_1 =0.4667) and the crack depth is 10 *mm* ($\bar{\gamma}_1 = 0.50$), the lowest measured three natural frequencies are: 171 Hz, 987 Hz and 3034 Hz [12]. It can be observed that the first two measured natural frequencies are very close to our designated natural frequencies.

Example B: For another comparison, the case is of a cantilever beam with one single-side open crack. The beam parameters are: length L=580 *mm*, width B=12.7 *mm*, height H=12.7 *mm*, Young's modulus of elasticity E=2.06×10^{11} N/m^2, and density ρ=7800 Kg/m^3. The desired two natural frequencies of the beam are 29.0 Hz and 180.0 Hz. Using the method proposed in this article, two curves are obtained as shown in Fig. 6. The solution of these two curves is l_1 =0.383 and θ_1=0.1790 (refer to Fig. 6).

The non-dimensional crack depth corresponding to this value of crack flexibility is $\bar{\gamma}_1 = 0.715$ as shown in Fig. 7.

In the experiment in [6], the crack is located 230 mm from the fixed end (l_1 =0.3965) and the crack depth is 8.5 mm (a_1 =0.67). Using the experimental modal test, the measured transfer function is shown in Fig. 8 and the measured lowest three natural frequencies are: 28 Hz, 179 Hz and 502 Hz [6].

Table 1 and Table 2 show the comparisons for the soft tuning of the non-dimensional crack location and the non-dimensional crack depth and the experimental results for the above examples. It is also observed that the measured frequencies of the cracked beam system are very close to our tuning designated natural frequencies.

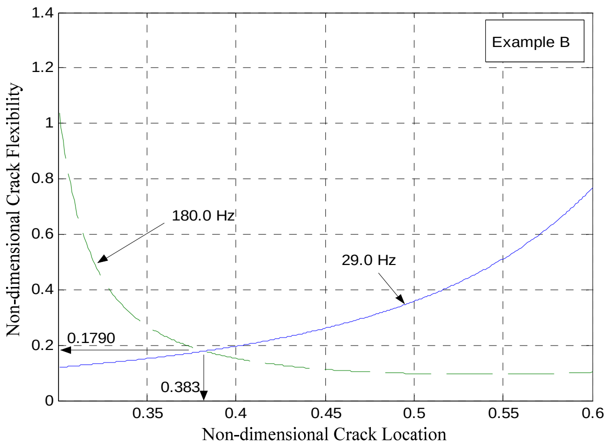

Fig. 6. The determination of the non-dimensional crack location l_1 and non-dimensional crack flexibility θ_1 using two tuning frequencies for *Example B*.

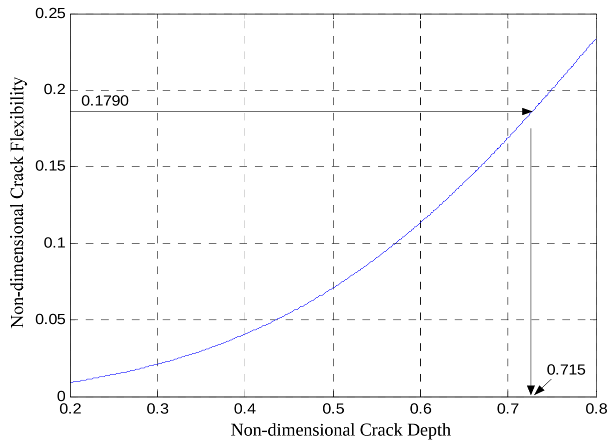

Fig. 7. The determination of the non-dimensional crack depth $\bar{\gamma}_1$ from the non-dimensional crack flexibility θ_1 for *Example B*.

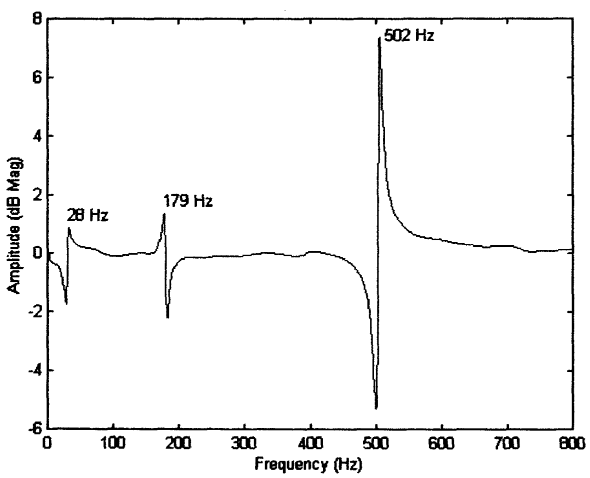

Fig. 8. Measured transfer function for a cracked cantilever beam of *Example B*

Table 1. Comparisons (*Example A*) of crack tuning for a cantilever beam using in ref [12]

	Tuned (designated)	Measured (validation)
Natural frequency (Hz)	174.0 988.9	171.0 987
Non-dim. crack location l_1	0.4670	0.4667
Non-dim. crack depth $\bar{\gamma}_1$	0.498	0.50

Table 2. Comparisons (*Example B*) of crack tuning for a cantilever beam using in ref [6]

	Tuned (designated)	Measured (validation)
Natural frequency (Hz)	29.0 180.0	28 179
Non-dim. crack location (l_1)	0.383	0.3996
Non-dim. crack depth ($\bar{\gamma}_1$)	0.715	0.67

If only one natural frequency is required to be tuned, the solution pair (l_1, θ_1) is not unique. When two natural frequencies are required, there is only one solution pair (l_1, θ_1). This means that two natural frequencies can be tuned for one crack. Moreover, when three natural frequencies are required, at least two cracks are needed for tuning this system. Theoretically, using the same method, although a little bit complicated, the solutions still can be found. Practically, the conditions for multi-frequencies tuning is not very common in engineering applications.

V. CONCLUSIONS

A technique for tuning the natural frequencies of beam structures to designated values is presented. The method utilizes a transfer matrix method to obtain an analytical form of the characteristic equation of the system. The natural frequencies are varied by tuning the crack extent and crack location of the beam structures. Results showed that multiple natural frequencies could be tuned simultaneously to the desired values and the method was experimentally validated.

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