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# Study of Loss Reduction Problem in Distribution Systems by Using Multi-objective Approach
# 應用多目標規劃法解決配電系統中減少線路損失問題之研究

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## Abstract

This study presents a multi-objective programming method to resolve the loss reduction problem in distribution systems. The problem formulation proposed herein considers different objective functions relating to minimizing the system power loss, the number of the switching operations, the deviation of the bus voltage, and the customer risks. Meanwhile, the operation constraints, the radial structure of the network configuration and power supplied for all loads, are included. These objective functions are modeled with fuzzy sets to evaluate their imprecise nature, then the optimization problem is solved by evolutionary programming techniques. Simulation results on the Tai-power system demonstrate the effectiveness of the solution algorithm.

**Key Words:** Loss reduction, multi-objective programming, evolutionary programming, power flow analysis.

## 摘　要

本文針對配電系統提出一套多目標規畫方法來解決線路損失最小化問題，此問題涵蓋的目標函數計有線路損失最小化、連絡開關操作次數、匯流排電壓變動量、以及客戶風險，同時要也要在各種不同負載下維持線路輻射狀結構操作限制·上述目標函數皆以模糊歸屬函數來表示以符合其非精準特性，然後利用進化規畫法技術來尋求最佳化解答·本文藉由台電配電系統來做模擬，結果亦顯示本文所提規畫方法極為快速與正確·

**關鍵詞**：線路損失最小化、多目標規畫法、進化規畫法、電力潮流分析

## 1. INTRODUCTION

In modern distribution system, lots of power generation is wasted in the form of line loss. Planning a switch strategy to reduce the power losses is a critical task for dispatchers in a control center. In practice, a dispatcher recommends the procedure for loss reduction according to his or her practical experiences. With increasing complex nature of distribution system, determining these procedures is extremely difficult, particularly for new dispatchers. Therefore, a method must be developed to assist dispatchers in drawing up loss reduction plans.

Many approaches have been proposed to resolve the loss reduction problem from different perspectives. For instance, [1] used analytical approaches and derived a quadratic expression for system losses. In [2-4], heuristic algorithms were employed to minimize the power loss. [5] developed a global optimality condition of the problem and two solution algorithms. One obtains the optimal solution when the minimum is obtained for every feeder pair, the other is performed by moving open points one at a time. In [6], explicit loss reduction and line flow formulas were developed to facilitate efficient determination of the switching operations. Recently, AI-based approaches have been proposed for the problem and the results are encouraging. Such as expert system [7], neural network [8], simulated annealing [9], genetic algorithm [10,11] and evolutionary programming [12,13]. Because of the large search space, most AI algorithms can discover global optimum. In this paper, a multi-objective programming based on fuzzy is presented. The problem formulation proposed herein considers different objectives relating to minimizing (1) system power loss, (2)number of switching operations, (3)deviation of the bus voltage, and the risks of (4)feeders and (5)transformers. Meanwhile, the operation constraints, the radial structure of the network configuration and power supplied for all loads, are included. The proposed method adopts evolutionary programming (EP) owing to its appropriateness in solving the optimization problem [14,15]. The main features of the proposed algorithm are described as follows.

(1) Allows dispatcher to find a optimal solution.
(2) Identifies the reconfiguration plans quickly and effectively.
(3) Can be applied to large-scale distribution systems.
(4) Considers a more realistic problem formulation.

The rest of this paper is organized as follows. Section 2 describes a novel formulation of the loss reduction problem. Section 3, a multi-objective programming method in fuzzy notations is proposed. In section 4, we describe how to apply the implement technique of the proposed method to the loss reduction problem. Section 5 then demonstrates the effectiveness of the solution algorithm on a Tai-power distribution system. Conclusions are finally made in section 6.

## 2. PROBLEM FORMUTATION

Reconfiguration for loss reduction in distribution systems is to operate the existed network switches to reduce the system power loss. In this section, we formulate the loss reduction problem as to minimize the power losses, the number of switching operations, the deviation of the bus voltage, and the risks of the feeder and the transformer in conjunction with network operating constraints.

## 2.1 Objective Functions

***(1) Minimize the system power loss***

$$\text{Min} \quad f_1(\overline{X}) = \sum_{i=1}^{N_l} r_i \frac{p_i^2 + q_i^2}{v_i^2} \qquad (1)$$

where, $r_i$, $p_i$, $q_i$, are the resistance, real power, reactive power of branch i, and $v_i$ is the voltage on bus i. $N_l$ is the total branches number. $\overline{X}$ is the state variable which is a vector consisting of all the opened switches in the system. Every $\overline{X}$ represents a new configuration of the system. $f_1(\overline{X})$ is system power loss of the configuration under the state $\overline{X}$.

***(2) Minimize the number of switching operations***

$$\text{Min} \quad f_2(\overline{X}) \qquad (2)$$

$f_2(\overline{X})$ denotes the number of switching operations under state $\overline{X}$ corresponding to the original system.

***(3) Minimize the deviation of the bus voltage***

$$\text{Min} \quad f_3(\overline{X}) = \max_i |V_i - 1.0|, \ \text{i} = 1,2,.\ N_b \qquad (3)$$

where, $N_b$ is the total number of the buses, $V_i$ is represented in per unit, $f_3(\overline{X})$ represents the maximal deviation of bus voltage in the considered system.

***(4) Minimize the risk index of feeder***

$$\text{Min} \quad f_4(\overline{X}) = \max\{\frac{I_{iLoad}}{I_{iRate}}\}, \ \text{i} = 1,2,\ldots N_l \qquad (4)$$

where, $I_{iLoad}$ and $I_{iRate}$ are the load current and rated current of branch i respectively, $f_4(\overline{X})$ is the risk index of feeder in the considered system.

***(5) Minimize the risk index of transformer***

$$\text{Min} \quad f_5(\overline{X}) = \max\{\frac{tr_{iLoad}}{tr_{iRate}}\}, \ \text{i} = 1,2,..\ N_t \qquad (5)$$

where, $N_t$ is the total number of the transformers, $tr_{iLoad}$ and $tr_{iRate}$ are the load current and rated current of transformer i, respectively, $f_5(\overline{X})$ represents the risk index of transformer loading.

## 2.2 Constraints

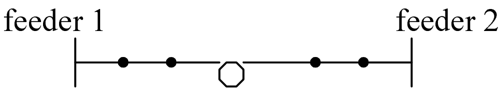
In the formulation of the problem, two constraints are included.

(1) The radial structure of network must be remained.
(2) All loads must be supplied.

Based on [5], the reconfiguration can be achieved by moving the open switch position in a feeder pair. Thus we simply chose any one of three actions of the open switch in a feeder pair randomly as follows.

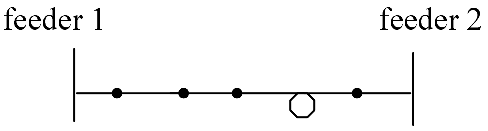
(a) Keep unchanged (original)
(b) Forward shift
(c) Backward shift

The illustration is shown by Fig. 1. The three actions belong to the methods known as “branch exchange techniques” and guarantee the conditions in constraints (1), (2).

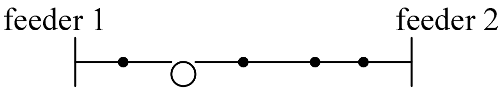


(a) Original feeder pair

○ -opened switch　● -closed switch



(b) Forward shift



(c) Backward shift

Fig. 1 Three actions of the opened switch in a feeder pair

## 3. MULTI-OBJECTIVE PROGRAMMING IN FUZZY NOTATIONS

Consider a multiple objective problem as the following form:

$$\text{Min} f_i(\overline{X}), \quad i=1,2,\ldots,m \qquad (6)$$

subject to

$$g_j(\overline{X})=0, \quad j=1,2,\ldots,k \qquad (7)$$

where $f_i(\overline{X})$ are m distinct objective functions of the decision vector $\overline{X}$, and $g_j(\overline{X})=0$ are k different constraints. Fundamental to the multiple objective problem is the noninferior solution [16]. Qualitatively, a noninferior optimal solution of the multiple objective problems is one where any improvement of one objective function can be achieved only at the expense of another. Usually, noninferior optimal solutions consist of an infinite number of points, and some kinds of subjective judgement should be add to the quantitative analysis by the decision maker (e.g. dispatchers in the control center decide the loss reduction procedure for multiple objectives). In this section, we propose the multi-objective programming in fuzzy notations to determine the optimal solution of the decision-maker.

### 3.1 Fuzzy Membership Function

Considering the imprecise nature of each objective function, we formulate these objective functions as fuzzy sets. In general, fuzzy set is represented by a membership function $\mu_{fi}(\overline{X})$. The higher the value of the membership function the greater satisfaction with the solution will be. The membership function consisting with a lower and upper bound value together with a strictly monotonically decreasing and continuous function. Figure 2 illustrates the graph of the possible shape of strictly monotonically decreasing membership function. To elicit a membership function $\mu_{fi}(\overline{X})$ for each objective function $f_i(\overline{X})$, we first decide the lower and upper bounds, $f_i^{min}(\overline{X}), f_i^{max}(\overline{X})$ of each objective function under given constraints. Then, we determine a strictly monotonically decreasing and continuous function $h_i(f_i(\overline{X}))$ which can be linear or nonlinear. For a minimizing problem, a membership function is defined by

$$u_{fi}(\overline{X}) = \begin{cases} 1 \ \ or \to 1, & if, \ f_i^{\min} \ge f_i(\overline{X}) \\ h_i(f_i(\overline{X})), & if, \ f_i^{\min} \le f_i(\overline{X}) \le f_i^{\max} \\ 0 \ \ or \to 0, & if, \ f_i(\overline{X}) \ge f_i^{\max} \end{cases} \qquad (8)$$

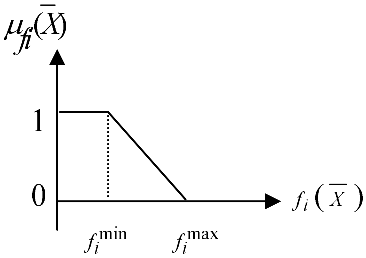


Fig. 2. An example of membership function.

### 3.2 Multi-objective Programming

To generate a candidate for the satisfied solution of the formulated problem, the decision-maker must specify his expected value of the achievement of the membership functions. The expected value is a real number between [0, 1] represented the level of importance of each objective function. After giving the dispatcher's expected values $\overline{\mu_{fi}}$, the following minimax problem is solved to generate the optimal solution, which is closed to his requirements.

$$\underset{X \in S}{Min}\ \{ \underset{i=1,2,\ldots..m}{Max}\ [\overline{\mu_{fi}} - \mu_{fi}(\overline{X})]\} \tag{9}$$

where, S is the vector space of $\overline{X}$, and $m$ represents the number of total objective functions. We can now describe the method of the optimization technique.

Step 0: Input data.

Step1: Decide the upper and lower bound for every objective function, $f_i^{\min}$ and $f_i^{\max}$, and elicit the strictly monotonically decreasing function to formulate the membership functions, $\mu_{fi}(\overline{X})$.

Step 2: Decide the expected value of each objective function, $\overline{\mu_{fi}}$, for i = 1, 2, … $m$.

Step 3: Apply EP (described in the next section) to solve the minimax problem.

Step4: Output the optimal solution, $\overline{X}$.

## 4. SOLUTION ALGORITHM FOR MULTI-OBJECTIVE LOSS REDUCTION PROBLEM

In this section we introduce the technique to implement the proposed method to solve the loss reduction problem.

### 4.1 Mathematical model of objective function

Five objective functions considered in the loss reduction problem are represented in fuzzy sets with the lower and upper bounds as well as strictly monotonically decreasing functions. The five different objective are to minimize (1) the system power loss, (2) the number of the switching operations, (3) the deviation of the bus voltage, and the risks of (4) feeders and (5) transformers. Figure 3 to 6 schematically depict these objective functions. Table 1 shows the critical parameters of the objective functions.

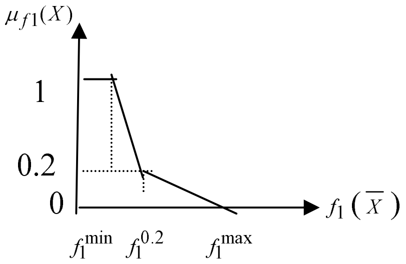


Fig. 3 Membership function $\mu_{f1}(\overline{X})$

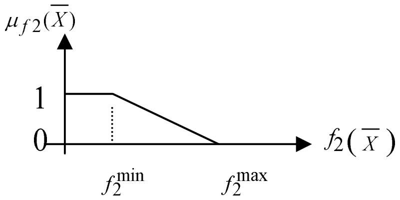


Fig. 4 Membership function $\mu_{f2}(\overline{X})$

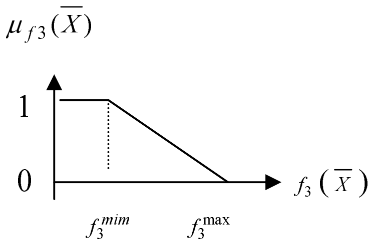


Fig. 5 Membership function $\mu_{f3}(\overline{X})$

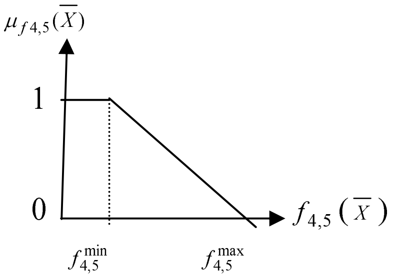


Fig. 6 Membership function $\mu_{f4,5}(\overline{X})$

Table 1. Parameters of Objective Functions

| Objective function | Parameter |
|---|---|
| Total power losses of lines | $f_1^{\min}=0.65\,f_i(\overline{X_0})$, $f_1^{0.2}=f_i(\overline{X_0})$<br>$f_1^{\max}=2\,f_i(\overline{X_0})$<br>$f_2^{\min}=0$, $f_2^{\max}=26$ |
| Numbers of the switching operation | $f_3^{\min}=0.05\text{pu}$, $f_3^{\max}=0.1\text{pu}$ |
| Deviation of the bus voltage | $f_4^{\min}=1.0$, $f_4^{\max}=1.25$ |
| Feeder risk | $f_5^{\min}=1.0$, $f_5^{\max}=1.25$ |
| Transformer risk | |

Remark: the lower and upper bound $f_i^{min}$ and $f_i^{max}$ depend on the constraints of the considered problem.

## 4.2 Evolutionary programming

EP was developed to be a global optimization algorithm with multi-path search according to the processes observed in natural evolution. Evolution is a process that operates on artificial chromosome. Each chromosome consists of genes and links to a fitness that represents a measurement of the worth of the chromosome. In EP, the generation selection is performed by mutation and competition, not the reproduction, crossover and mutation in GA. In the process of evolution, parents are replaced by their better offspring. The best individual in the final population can be a highly evolutionary solution to the problem.

For the problem of loss reduction in a distribution system, EP is implemented to obtain the best configuration of the network. Each state variable, $\overline{X}$ represents a chromosome in which each gene represents an opened switch. For example, if the opened switches in a given network are $s_1$, $s_2$, and $s_3$, then the chromosome is given by $\overline{X}=[s_1, s_2, s_3]$. For the min-maximum problem, the fitness of $\overline{X}$ is described as follows according to Eq. (9).

$$fitness=\frac{1}{1+obj(\overline{X})} \qquad (10)$$

and $obj(\overline{X})=\underset{i=1,2,\ldots.m}{Max}[\overline{u_{fi}}-\mu_{fi}(\overline{X})]$ ……(11)

For a given $\overline{X}$, the more the fitness is high, the more the solution is close to the optimum. The flow chart of EP is shown in Fig. 7 and illustrated as follows.

***Input data***

Input EP parameters including the length of the chromosome string and the population size, N.

***Initialization***

The original state vector, $\overline{X0}$ and its derivations (moving the open switches) can be used as the first population.

***Competition***

The chromosomes with higher fitness values have the more opportunity to survive or breed in next generation. Thus the number of offspring, $n_i$ for each chromosome i is given by

$$n_i = G[N \times \frac{C_i}{\sum_{i=1}^{N} C_i}] \qquad (12)$$

where, $C_i$ represents the fitness value of chromosome i, G[x] round the elements of x to the integer.

***Elite mutation***

According to the moving rules of the opened switches discussed in Sec. 2.2, the $n_i$ offspring are mutated from chromosome i. If the sum of $n_i$ is less than N, the deficits are complemented by derivations of the best chromosome.

***Stop criterion.***

Evaluate all the fitness values of chromosomes in the population. If the best chromosome keeps unchanged after a preset iteration's number, then output the solution. Otherwise go back to competition procedure.

**Remark:**

The best chromosome in the parent generation is kept in children population. That can avoid the divergence of the EP.

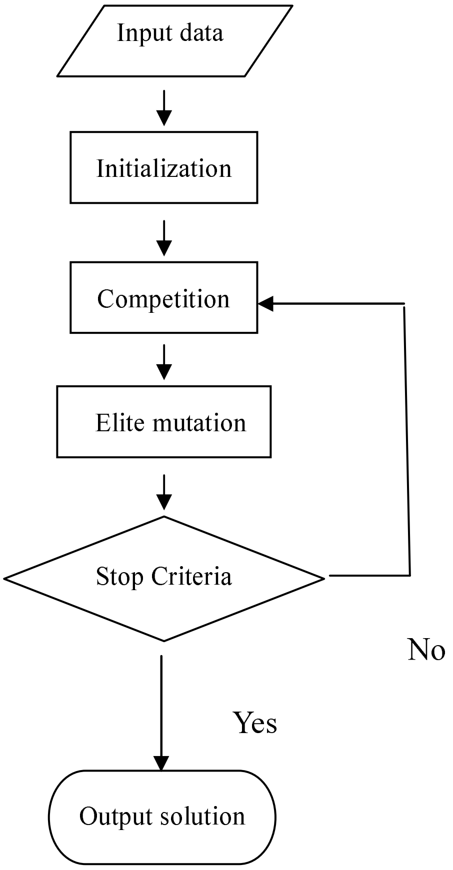
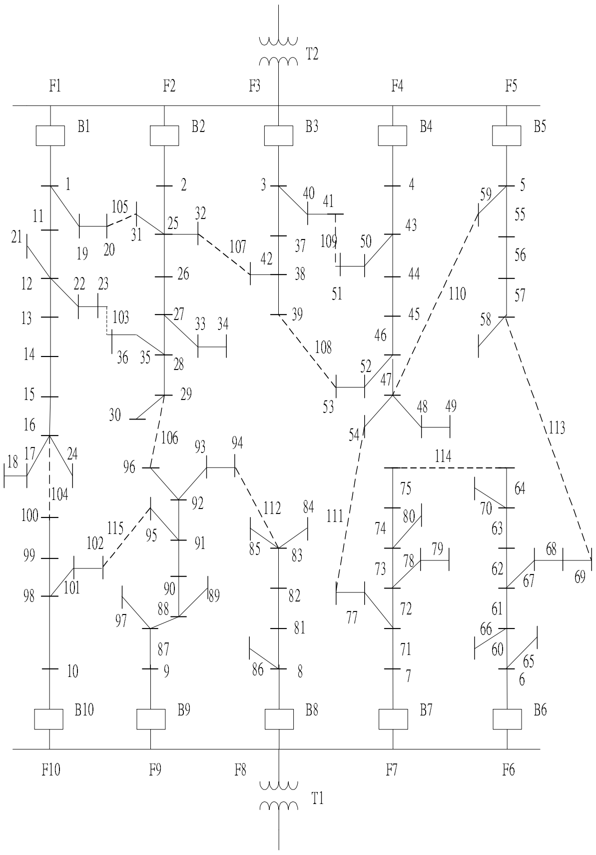


Fig. 7 Flow chart of the EP

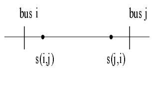
# 5. SIMULATION RESULTS

## 5.1 Example Illustration

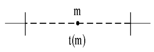
Based on the proposed algorithm, a time-sharing computer program is implemented in C++ with man-machine interactive procedures. A distribution system of the Tai-Power Company is tested by the proposed method. This system includes 2 transformers, 10 feeders, 102 branches, 13 tie lines, 102 buses and 217 switches. Fig. 7 illustrated the network structure of the system. The loss reduction problem is to decide positions of the opened switches for minimizing the system power loss. That is there are $C_{13}^{217}$ possible combinations in the solution space. The searching space is so large that most of optimum algorithms can not effectively solve the problem. The parameters of EP used in this system are described as follows: elements number of the chromosome string: 13 and population size: 60.



(a) system configuration



(b) switch location



(c) tie_switch location

Fn : No. of feeder
Bn : No. of breaker
□: Breaker
—: Feeder line
......: Tie line
m : No. of tie switch
......

Fig. 8. Network structure of the testing system.

## 5.2 Results

From above test cases, these results in Table 2 reveal that the proposed method can be implemented in practical system. In addition, the run time is fast for application in on-line system. For all cases, the reconfiguration plan was obtained under 71 seconds on a Pentium-CELERON 300A PC.

**Table 2. Results of the test case.**

| Loading level | Low | | Medium | | Heavy | |
|---|---|---|---|---|---|---|
| | Before reconfig | After reconfig | Before reconfig | After reconfig | Before reconfig | After reconfig |
| Power loss (kW) | 205.27 | 104.49 | 325.66 | 164.45 | 476.29 | 238.56 |
| Reduce loss rate (%) | --- | 49.1% | --- | 49.5% | --- | 49.9% |
| No. of Switching operation | --- | 24 | --- | 26 | --- | 26 |
| Max. of deviation of bus voltage (pu) | 0.039 | 0.020 | 0.050 | 0.025 | 0.060 | 0.030 |
| Min. of the margin loading among feeders (%) | 56.0% | 72.2% | 44.6% | 65.1% | 33.0% | 58.0% |
| Min of the margin loading among transformers (A) | 63.5% | 65.2% | 54.1% | 56.4% | 44.6% | 47.5% |
| CPU time (second) (2 runs) | --- | 70 | --- | 70 | --- | 71 |
| Location of the tie switch (# of bus) | T(103)<br>T(104)<br>T(105)<br>T(106)<br>T(107)<br>T(108)<br>T(109)<br>T(110)<br>T(111)<br>T(112)<br>T(113)<br>T(114)<br>T(115) | S(22,12)<br>S(16,15)<br>S(19,1)<br>T(106)<br>S(38,42)<br>S(52,46)<br>S(43,50)<br>S(46,45)<br>S(47,54)<br>S(92,91)<br>S(67,68)<br>S(63,64)<br>S(91,95) | T(103)<br>T(104)<br>T(105)<br>T(106)<br>T(107)<br>T(108)<br>T(109)<br>T(110)<br>T(111)<br>T(112)<br>T(113)<br>T(114)<br>T(115) | S(22,12)<br>S(16,15)<br>S(19,1)<br>S(96,92)<br>S(38,42)<br>S(52,53)<br>S(43,50)<br>S(46,45)<br>S(47,54)<br>S(92,91)<br>S(67,61)<br>S(63,64)<br>S(91,95) | T(103)<br>T(104)<br>T(105)<br>T(106)<br>T(107)<br>T(108)<br>T(109)<br>T(110)<br>T(111)<br>T(112)<br>T(113)<br>T(114)<br>T(115) | S(22,23)<br>S(16,15)<br>S(38,42)<br>S(96,92)<br>S(19,1)<br>S(52,53)<br>S(43,50)<br>S(46,45)<br>S(47,54)<br>S(92,91)<br>S(67,68)<br>S(63,64)<br>S(91,95) |

## 6. CONCLUSIONS

In this paper, a multi-objective programming method based on fuzzy is presented to solve the loss reduction in a distribution system. Five different objectives considered herein are to minimize the system power loss, number of switch operations, bus voltage deviation, and risks of feeder and transformer in conjunction with network constraints. Owing to the multi-path search ability of EP to solve the problem with nonlinear and non-differentiable objective functions, this investigation applies EP to our solution algorithm to derive the optimal solution. Finally, the proposed method has been implemented and tested on practical distribution system of Tai-Power. Based on the test results, the proposed method can find the optimal solution fast and correctly even in a system with large search space.

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