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變動抽樣間隔全距管制圖在 Gamma 分配資料的監控績效評估

Range Control Charts with Variable Sampling Interval under Gamma Distribution Assumption

Pei-Hsi Lee[1] Chih-Ping Chiang[2]

摘 要

在常態資料假設下，變動抽樣間隔全距管制圖能快速偵測出製程標準差的變異，然而，在真實產業製程中，觀測值未必都會完全符合常態，也可能會呈現偏態機率分配，gamma分配是生產製程中最常發生一種偏態機率分配，本研究秀出了變動抽樣間隔全距管制圖在gamma分配假設下的監控績效，並與傳統的修華特全距管制圖進行比較，從比較結果發現到，不論是常態或gamma分配資料，變動抽樣間隔全距管制圖都會有較佳的製程監控績效，不過，當母群體資料遠離常態假設時，變動抽樣間隔全距管制圖的偵測能力就會變得不敏銳。

關鍵詞：全距管制圖、適應性管制圖、統計性設計、馬可夫鏈方法、gamma 分配

ABSTRACT

Range control charts with variable sampling interval (VSI *R* chart) have better performance in monitoring the variation of process standard deviation under normality. However, the process observations may violate normality and follow a gamma distribution as a skewed distribution. This paper presented the performance of VSI *R* chart under gamma distribution assumption. A comparative study shows that VSI *R* chart has better performance than Shewhart *R* chart under normal and gamma distribution, but VSI *R* chart will become un-sensitivity when the population is far away from normality.

Keywords: Range control chart; adaptive control charts; statistical design; Markov chain method; gamma distribution

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1. Introduction

Shewhart $\overline{X}$ chart always assumes that the process standard deviation remains unchanged and then signals the mean variation. However, in real processes, the standard deviation may occur shifts. When the process standard deviation stays on an initial value, the process is in-control state, but if the standard deviation had shifted, the process state is out-of-control. The shift occurrence of the process standard deviation will cause the decreasing of the process capability and the increasing of the defective rate. If this shift of process standard deviation or variance can not be quickly detected, it is easy to cause higher loss.

Shewhart range chart (*R* chart) and Shewhart *s* chart (*s* chart) can monitor variation of the process standard deviation or variance. An *R* chart monitors variation of standard deviation with sample range which is the difference between the largest and smallest observations of a sample of size *n*. An *R* chart is easer than an *s* chart in set up control limits, but its relative efficiency will be less than 85% of *s* chart when sample size $n > 10$ often employs on control charts. In addition, *R* chart and *s* chart are un-sensitivity in detecting small shifts.

Many previous studies applied ideas of adaptive control charts to improve efficiency of Shewhart-type charts in detecting small process shifts. Adaptive control charts include: variable sampling intervals (VSI); variable sample sizes (VSS); variable sample sizes and sampling intervals (VSSI); variable sample sizes and control limits (VSSC) and variable parameters (VP) control charts, and their applications have successfully increased the sensitivity of $\overline{X}$ chart in detecting small shifts[3,7-11,17,18, 21-25,31,32]. Although Costa[9,10] and Chou et al.[6] had performed the designs of joint adaptive $\overline{X}$ and *R* charts and indicated joint adaptive $\overline{X}$ and *R* charts have the best efficiency when both mean and variance occur small shifts. However, previous literatures never clear evaluated performance of adaptive *R* charts without jointing other $\overline{X}$ charts.

Statistical process control (SPC) methods are often based on two assumptions: first, the sample observations are statistically independent; second, the process observations follow a normal distribution. Violation of the normality assumption can cause a serious problem when applying control charts in process monitoring. Burr[1] notes that the usual normal theory based control limit constants are very robust to the non-normality assumption and can be employed unless the population is extremely non-normal. Kao & Ho[13] examined the performance of an *R* chart and found the *R* chart is robust to non-normality. Torng & Lee[30] presented the performance of Tukey's control chart under non-normality. Chen & Cheng[4] and Chen[5] studied respectively the designs of $\overline{X}$ chart and VSI $\overline{X}$ chart under non-normality from the cost viewpoint. Lin & Chou[14-17] had presented the performances of the adaptive control charts under non-normality. Torng & Lee[29] had examined performances of double sampling $\overline{X}$ charts (DS $\overline{X}$ charts) and adaptive control $\overline{X}$ charts and found DS $\overline{X}$ charts had good performance as well as adaptive control $\overline{X}$ charts under non-normality.

VSI chart changes sampling interval of Shewhart chart to monitor process. When the sample point is close to the center line, the chart is to use the long sampling interval to take sample. If the sample point heavily deviates the center line but does not fall out the control limits, short sampling interval (h_2) is used to monitor the process. VSI chart can shorten the time to signal a process variation, and it is easer to use than other adaptive charts. In real industry, the process observations always follow a skewed distribution. In this study, we selected a gamma distribution as the skewed distribution to examine and compare the performance of VSI *R* charts under gamma distribution assumption with Shewhart *R* chart.

2. Range control charts with variable sampling interval

A random sample of size *n* is taken from the population and the range of this sample is defined as $R = x_{\max} - x_{\min}$.

A VSI *R* chart uses two different sampling intervals. When the sample range *R* is close to the average range, VSI *R* chart is to use the long sampling interval (h_1) to monitor the process standard deviation. However, if the sample range *R* heavily deviates the average range but does not fall out the control limits, short sampling interval (h_2) is used to monitor the process standard deviation.

A VSI *R* control chart as figure 1 employs warning limits and control limits to divide the chart into three regions: in-control region, warning region and out-of-control region. Let respectively *k* and *w* be as the control limit and the warning limit coefficients of VSI R control chart, the center line, control limit and warning limit of VSI R control chart are

$$UCL = [d_2(n) + kd_3(n)]\sigma_0$$
$$UWL = [d_2(n) + wd_3(n)]\sigma_0$$
$$CL = d_2(n)\sigma_0 \quad (1)$$

where σ_0 is the standard deviation of an in-control process; CL, UCL and UWL are center line, the upper control limit and upper warning limit; $d_2(n)$ is a coefficient for sample size n, that can be estimated by $E(R)/\sigma_0$; $d_3(n)$ is an another coefficient for sample size n, it can be obtained by $\sqrt{V(R)}/\sigma_0$. Tippett[28], Mahoney[20] and Kao & Ho[13] had presented the calculations of $d_2(n)$ and $d_3(n)$ when the population was following normal distributions and non-normal distributions.

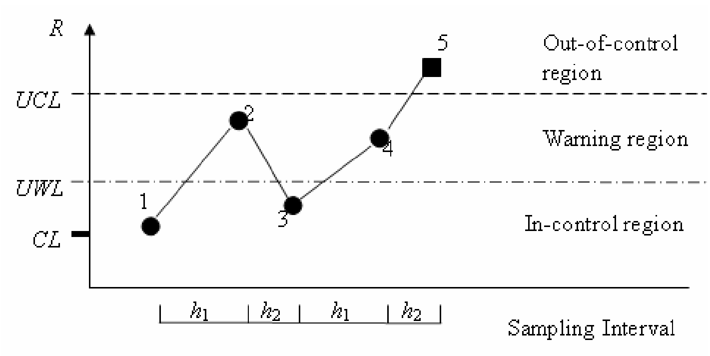


Figure 1 A VSI R char and its control procedure

Assumed standard deviation of the initial process is an in-control state. First sampling will take a sample size n to calculate the sample range R and plot the sample range R on the control chart. From the 1st sample point falls in in-control region, interval of 2nd sampling will be h_1, and then 2nd sample range is plotted on chart. The 2nd sample range falls in warning region, so the 3rd sampling will use short interval h_2, and plot the sample range on chart. The 3rd sample point falls in in-control region, and then the next sampling uses interval h_1 to take sample. From the 4th sample point falls in in-control region, the 5th sampling will use h_2 to take sample for monitoring of standard deviation. For 5th sampling result, the sample range falls out the control limit, and then the chart indicates that the process standard deviation is out-of-control. Assumed n_0, h_0 and k_0 are the sample size, sampling interval and control limit coefficient of a Shewhart R chart, respectively, when the $h_1 = h_2 = h_0$, $n = n_0$ and $w = k = k_0$, the VSI R chart will become a Shewhart R chart.

3. Design of VSI R charts under gamma distribution assumption

3.1 d_2 and d_3 values of gamma distribution

The gamma distribution, denoted by G(a,b), has the probability density function

$$f_g(x|a,b) = \frac{x^{a-1}\exp(-x/b)}{b^a\Gamma(a)}, \quad x > 0,\ a,b > 0 \quad (1)$$

where the a and b are respectively the shape and scale parameters, and $\Gamma(\bullet)$ is a gamma function. The mean and variance of a gamma distribution are ab and ab^2, respectively. The values of d_2 and d_3 of gamma distribution can be obtained with a simulation approach, and the simulation procedure was coded with Matlab R2007a. This simulation procedure is as follows:

(1) Generate n data from G(a,b) and calculate the sample range R.

(2) Repeat step (1) 100,000 times to obtain 100,000 sample range R.

(3) Calculate the expected value μ_R and standard deviation σ_R of 100,000 sample ranges, respectively.

(4) Calculate values of d_2 and d_3 by $\mu_R / \sqrt{ab^2}$ and $\sigma_R / \sqrt{ab^2}$, respectively.

3.2 Performance indicators

Tagaras[27] presented several statistical indicators of adaptive control charts, such as:

1. Average Time to Signal (ATS), which is defined as the expected value of the time from the start of the process to the time when the chart indicates an out-of-control signal. ATS represents a measure of the false alarm rate.

2. Adjusted Average Time to Signal (AATS), which is defined as the expected value of the time from the occurrence of an assignable cause to the time when the chart indicates an out-of-control signal. AATS represents the detection time of control chart when the process mean has shifted.

Assumed the initial state of a process is in control, and the process standard deviation is $\sigma = \sigma_0$. When an assignable cause occurs, the process standard deviation shifts from the $\sigma = \sigma_0$ to $\sigma = \sigma_1 = \gamma\sigma_0$, where γ is shift size coefficient and can be estimated by $\gamma = \sigma_1 / \sigma_0$. A case of γ = 1 indicates the process standard deviation is an in-control state. If γ > 1, the $\sigma_1 > \sigma_0$. That is to say the process standard deviation had shifted, and the quality of products had become inferiority. For γ < 1, the $\sigma_1 < \sigma_0$, it indicates the quality of products had be superior. In general, a case of γ < 1 can not occur in real process unless the process had already been improved. Therefore, control charts only detect shifts of γ > 1.

Lin & Chou[17] provided an easy approach to calculate performance indicators of adaptive control charts, and their approach can also be applied on VSI *R* control charts. The ATS and AATS of a VSI *R* control chart can be calculated from Markov chain method, which includes the following three transition states:

State 1. The process is in control and the sample point falls in the in-control region.
State 2. Process is in control and the sample point falls in the warning regions.
State 3. The process is out-of-control.

Let **Q** be state transition probability matrix

$$\mathbf{Q} = \begin{bmatrix} q_{11}(\gamma) & q_{12}(\gamma) \\ q_{21}(\gamma) & q_{22}(\gamma) \end{bmatrix} \quad (2)$$

where $q_{ij}(\gamma)$ indicates that when a shift γ is given, the probability that the sample range falls in the *j*th region of the *i*th control chart. Here, *j*=1 is the in-control region, and *j*=2 is the warning region. If the *i* = 1, using the long sampling interval (h_1) monitors the process, and when the *i* = 2, the short sampling interval (h_2) will be used in process monitoring. Calculations of four elements of **Q** are as following:

$$q_{11}(\gamma) = q_{21}(\gamma) = P(R \le UWL | \sigma_1 = \gamma\sigma_0) = F\left(\omega = \frac{UWL}{\gamma\sigma_0}\right) = F\left[\omega = \frac{d_2(n) + wd_3(n)}{\gamma}\right] \quad (3)$$

$$\begin{aligned} q_{12}(\gamma) = q_{22}(\gamma) &= P(UWL \le R \le UCL | \sigma_1 = \gamma\sigma_0) = F\left(\omega = \frac{UCL}{\gamma\sigma_0}\right) - F\left(\omega = \frac{UWL}{\gamma\sigma_0}\right) \\ &= F\left[\omega = \frac{d_2(n) + kd_3(n)}{\gamma}\right] - F\left[\omega = \frac{d_2(n) + wd_3(n)}{\gamma}\right] \end{aligned} \quad (4)$$

where ω is relative range, given by $\omega = R/\sigma$; $F(\omega)$ is the cumulative distribution function of ω, assumed process observations are following a gamma distribution with parameters a and b, and then the $F(\omega)$ is

$$F(\omega) = n\int_{-\infty}^{\infty} f_g(x|a,b)\left[F_g(x+\omega|a,b) - F_g(x|a,b)\right]^{n-1} dx \tag{5}$$

where n is sample size, $f_g(\bullet|a,b)$ and $F_g(\bullet|a,b)$ are the probability density function and cumulative distribution function of a gamma distribution with parameters a and b, respectively.

Let $\mathbf{h} = [h_1\ h_2]$ be sampling interval vector, $\mathbf{I}$ be an identity matrix with order 2 and $\mathbf{r}^{\mathrm{T}} = [r_1\ r_2]$ be defined as a steady-state probability vector. When the process is in control, the probability in the in-control and warning regions of the control chart can be used to calculate $\mathbf{r}^{\mathrm{T}}$. Let $\mathbf{P}$ be the probability matrix of an in-control process as in the following:

$$\mathbf{P} = \begin{bmatrix} p_1 & 1-p_1 \\ p_2 & 1-p_2 \end{bmatrix} \tag{6}$$

where p_i will be conditional probability: the probability that the given sample range is not falling out the control limits but falling in the central region. Calculations of p_1 and p_2 is as following

$$p_1 = p_2 = \frac{P(R \le UWL)}{P(R \le UCL)} = \frac{F(\omega = UWL/\sigma_0)}{F(\omega = UCL/\sigma_0)} = \frac{F[\omega = d_2(n) + wd_3(n)]}{F[\omega = d_2(n) + kd_3(n)]} \tag{7}$$

The calculation of $\mathbf{r}^{\mathrm{T}}$ will be

$$\mathbf{r}^{\mathrm{T}} = \begin{bmatrix} \dfrac{p_2}{1-p_1+p_2} & \dfrac{1-p_1}{1-p_1+p_2} \end{bmatrix} \tag{8}$$

The ATS of VSI R control chart can use Markov chain method to calculate as following:

$$\mathrm{ATS} = \mathbf{r}^{\mathrm{T}}(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{h} \tag{9}$$

If the position of the shift within the sampling interval is uniformly distributed, and the another indicator AATS will be

$$\mathrm{AATS} = \tilde{\mathbf{r}}^{\mathbf{T}}\left[(\mathbf{I}-\mathbf{Q})^{-1}\mathbf{h} - 0.5\mathbf{h}\right] \tag{10}$$

where

$$\tilde{\mathbf{r}}^{\mathbf{T}} = \begin{bmatrix} \dfrac{p_2h_1}{p_2h_1+(1-p_1)h_2} & \dfrac{(1-p_1)h_2}{p_2h_1+(1-p_1)h_2} \end{bmatrix}$$

The sampling interval $E(h|\gamma = 1)$ of VSI R control chart for an in-control process are

$$E(h|\gamma=1)=\text{ATS}(\gamma=1)/\text{ARL}(\gamma=1) \tag{11}$$

3.3. Statistical design model

The four design parameters of a VSI R chart are chosen taking into account a design model with the aim of making it comparable with R chart and other control charts. The objective function and constraints of design model that will be set for the design of a VSI R chart are: (1) that its in-control ATS takes a standard value τ, and 370.4 is always used to be this standard value; (2) that its expected sample size and sampling interval are equal to the values of a Shewhart R chart, respectively; and (3) that its out-of-control AATS value has to be minimized. Therefore, the design model is the following:

$$\begin{aligned} &Min \quad AATS(\gamma>1) \\ &s.t. \quad ATS(\gamma=1)=\tau \\ &\qquad E(h|\gamma=1)=h_0 \\ &\qquad h_1>h_0>h_2>0 \\ &\qquad k>w>0 \end{aligned} \tag{12}$$

The parameter k will be equal to the control limit coefficient k_0 of a Shewhart R chart. The decision variables of this design model only include three parameters: w, h_1 and h_2.

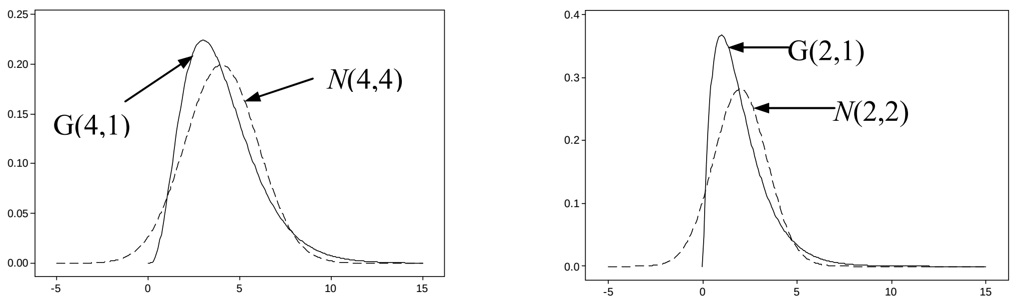
Before solving of two models, the shift size coefficient γ and sample size n must be determined first. This study assumes γ = 1.1 and n = 3 and 5 for solving in view of fast detecting small shifts. Matlab optimization toolbox applies a nonlinear constrained optimization algorithm for solving of nonlinear models. Matlab optimization toolbox had been used to determine the parameters of control charts[12,19,29]. The performance indicators were coded with Matlab R2007a, and optimization toolbox is then applied to solve design parameters of VSI R charts.

4. Comparison and discussion

4.1. Choose the parameters of gamma distributions

In this paper, we refer to Stoumbos & Reynolds[26], Calzada & Scariano[2], Lin & Chou[17] and Torng & Lee[29,30], and choose a = 4, 2 and a fixed b = 1 for the gamma distribution.

Figure 2 shows the gamma distributions we have selected and their corresponding normal distributions that have the same mean and variance. When a increases, the gamma distribution gets closer to a normal distribution. Through the use of these two types of gamma distributions, we can understand the effect of skewness change on the performances of control charts. These values of d_2 and d_3 under gamma distribution assumption can be obtained with a simulation approach in section 3.1, and these values for sample size n = 3 and 5 are listed in table 1.



(a)　(b)

Figure 2. The probability density function for various gamma and normal distributions: (a) G(4,1) and N(4,4); (b) G(2,1) and N(2,2)

Table 1. These values of d_2 and d_3 under gamma and normal distribution assumptions for n = 3 and 5

	n = 3		n = 5	
	d_2	d_3	d_2	d_3
G(4,1)	1.6488	0.9586	2.2604	0.9642
G(2,1)	1.5983	1.0293	2.2065	1.0567
N(0,1)	1.1280	0.8530	1.6930	0.8880

4.2. A comparative study of VSI *R* charts

In this section, a comparative study is conducted to evaluate the performance of VSI *R* charts and Shewhart *R* chart (SR). The false alarm rate of each chart is set to be equal, and so is the in-control expected sampling interval of each chart, such that the comparison can be conducted under the same criteria. This study choose n = 3 and 5 and h_0 = 1 to be the criteria of in-control sample size and sampling interval, respectively, and choose γ = 1.1, 1.2, 1.3, 1.4, 1.5, 1.75 and 2 to measure the performance of control charts. The shift sizes $1 < \gamma \le 1.5$ were defined as small standard deviation shifts, and shift sizes $\gamma > 1.5$ were defined as large shifts.

Table 2 provides the optimal design parameters of VSI *R* charts and *R* chart (SR) for various population probability distributions and the performance of all control charts. The in-control ATS values of all control charts are approximate 370.4. Table2 shows that AATS values of VSI *R* charts are smaller than values of SR charts for all population probability distributions. AATS values of VSI *R* charts under gamma distributions are larger than the value under normality.

In summary, performance of VSI *R* chart is better than Shewhart *R* chart for detecting standard deviation shifts under gamma distributions. When the population is far away from normality, the VSI *R* chart and Shewhart *R* chart will become un-sensitivity in detection of standard deviation shift.

5. Conclusions

In this study, we chose gamma distributions to evaluate the performance of VSI *R* chart, and compare its performance with Shewhart *R* chart. From the evaluation of statistical performance, we have found VSI *R* chart is not robustness to non-normality, but it still has better performance than Shewhart *R* chart for all shift sizes. If the monitoring variable follows a skew distribution, the observations should be transformed to follow a symmetrical distribution as a normal distribution before applied VSI *R* chart monitors the observations for increasing the detecting ability of standard deviation shift.

Table 2 Values of the ATS and AATS of VSI *R* charts and Shewhart *R* charts

	n = 3						n = 5					
	N(0,1)		G(4,1)		G(2,1)		N(0,1)		G(4,1)		G(2,1)	
	SR	VSI	SR	VSI	SR	VSI	SR	VSI	SR	VSI	SR	VSI
n	3	3	3	3	3	3	5	5	5	5	5	5
k	3.363	3.363	9.557	9.557	6.647	6.647	3.237	3.237	10.026	10.026	6.791	6.791
w	-	0.647	-	4.360	-	2.471	-	0.500	-	4.980	-	2.690
h_1	1.00	1.29	1.00	1.10	1.00	1.10	1.00	1.37	1.00	1.10	1.00	1.10
h_2	-	0.10	-	0.10	-	0.10	-	0.10	-	0.10	-	0.10
γ	ATS											
1.00	370.64	370.38	370.20	370.59	370.44	370.49	370.38	370.38	370.16	370.70	370.16	370.16
	AATS											
1.10	134.18	124.10	172.42	166.12	194.30	187.82	113.42	99.99	157.24	149.78	185.58	177.64
1.20	61.67	52.88	92.45	85.43	112.15	104.50	46.27	35.95	78.21	70.26	101.57	92.46
1.30	33.48	26.79	55.09	48.76	70.19	62.96	23.06	15.97	43.80	37.37	60.44	52.09
1.40	20.51	15.45	35.62	30.21	46.95	40.53	13.27	8.35	26.89	21.22	38.64	31.45
1.50	13.74	9.84	24.56	19.99	33.16	27.57	8.49	4.98	17.75	13.13	26.22	20.15
1.75	6.57	4.34	11.90	10.20	16.59	12.65	3.79	2.08	7.92	5.09	12.12	8.13
2.00	3.98	2.56	7.03	4.89	9.92	7.04	2.23	1.30	4.42	2.60	6.83	4.12

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