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Structural Reliability Analysis of Arch Bridges Based on Neural Network

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Abstract—**The uniform design method is adopted in selecting the training samples of neural network, and a method to calculate the reliability based on it has been brought forward. The calculation of the reliability of concrete-filled steel tubular arch bridge is achieved with Jiefang Bridge in Guangzhou, as the engineering background.**

Keywords-uniform design; neural network; concrete-filled steel tubular arch bridge; reliability

I. INTRODUCTION

Over the past two decades, long-span concrete-filled steel tubular (CFST) arch bridges have been widely built in China. But its design guidance and specification based on the reliability theory has not yet been completely established, and the approach for the reliability assessment and calculation of arch bridges still need to be further studied. Based on the characteristics of neural networks, the uniform design method is used to select the training samples of neural network. The method to calculate the reliability based on uniform design and neural network has been brought forward, which is applied to calculate the reliability of arch bridge with long-span. The results suggest that this method can simulate the real response of the bridge structure. Moreover, it can effectively carry out the assessment of structural reliability.

II. RELATED THEORIES

A. Uniform Design

Uniform design introduced in [1] was put forward by Professor Fang Kaitai and Wang Yuan in 1978. Its mathematical theory is consistent distribution theory in number theory. Belonging to the scope of pseudo-Monte Carlo methods, this method combines number theory and multivariate statistical. Uniform design only considers that within the scope of the study the test points are evenly spread. Test points with uniform distribution of the statistical properties can be ensured, so that each level of each factor does only one test. Test points of two random factors are in the plane grid point and in each row and each column there is only one test case, which focuses on the test points evenly spread within the scope of the study to get the most information by the least tests. So the number of tests can be significantly reduced compared with orthogonal design. Therefore, uniform design is ideal for the situations of testing with multi-factor, multi-level and system models completely unknown.

This method develops some well-designed tables to carry out tests, taking into account the test points within test scope are uniform dispersed. The number of trials is small. It has high efficiency. The tables are named uniform design tables which are expressed as $\mathbf{U}_n\left(q^s\right)$. Where, $\mathbf{U}$ means the uniform design. n is number of tests. q is number of layers. s is factors. By using this experimental design method, the number of trials can be reduced. For example, to conduct a comprehensive test, there will be a total of q^s combinations when the test has s factors and each factor has q levels. Orthogonal design selects q^2 tests from these combinations. Yet, uniform design selects n test points by taking advantage of consistent distribution theory in number theory. Sometimes n equals to q .And these test points within the integral range spread very evenly and they are adequately close to integrand. Therefore, it is very convenient for a statistical model to be established by the computer. Uniform design has very prominent advantages.

B. Neural Network

Neural Network (NN) or Artificial Neural Network (ANN) introduced in [2] and [3] is an important branch of artificial intelligence, referring to applying a large number of simple computing units (i.e., neurons) to make an interconnection range of non-linear network system, which imitates the functions of information processing, storage and retrieval of human brain system to a certain extent and level. Hence, it has intelligent processing functions such as learning, memorizing and computing.

Neural network has a strong learning ability, and can be trained to predict unknown samples. Connections between neurons can have arbitrary forms. However, only the forward neural network is currently used in the field of structural reliability analysis, i.e., multi-layer perceptron (MLP). A typical neural network contains only one hidden layer which is used in regression analysis and shown in Fig 1.

At present, Back Propagation (BP) algorithm and the Levenberg-Marquardt (L-M) algorithm are more commonly used as learning algorithm in multi-layer forward network. Derivations of various algorithms are all based on minimum mean-square error criteria.

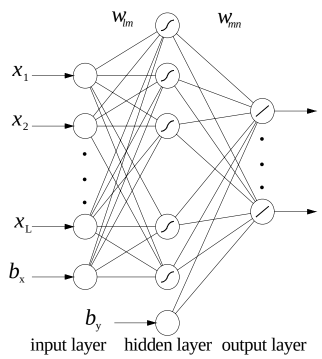

Figure 1. Multi-layer forward network

Suppose there is P training samples of a neural network. The total error can be expressed as:

$$E_A = \frac{1}{2}\sum_{p=1}^{P}\sum_{l=1}^{n}\left(z_l^{(p)} - \hat{z}_l^{(p)}\right)^2 \tag{1}$$

Where, $z_l^{(p)}$ is the practical output value l of the training sample p; $\hat{z}_l^{(p)}$ is the network output value l of the training sample p.

L-M training algorithm is simply introduced as follows.

We can see from the optimization theory that Newton's method usually converges much faster than the steepest descent method and has second termination. For the quadratic function, it can iterative converge to a stagnation point once. But Newton's method needs to calculate and store Hessian matrix. L-M algorithm is the adjustment of Newton's method. It doesn’t need to calculate the second derivative and it is very suitable for the neural network training with which performance index is a mean-square error. L-M algorithm is the most efficient in a variety of learning algorithms of multi-layer forward network.

In L-M algorithm, the weight $[k+1]$ can be expressed as:

$$\mathbf{w}[k+1] = \mathbf{w}[k] - \left[\mathbf{J}^T\mathbf{J} + \mu\mathbf{I}\right]^{-1}\nabla_w E \tag{2}$$

Where, $\mathbf{J}$ is the Jacobi matrix of the network output error to the weights; $\mathbf{I}$ is unit matrix; μ is a suitable non-negative value.

Compared with the standard Newton's method, L-M algorithm replaces the Hessian matrix of error function with $\mathbf{G} = \mathbf{J}^T\mathbf{J} + \mu\mathbf{I}$ approximately. Thus it avoids the trouble of seeking the second derivative.

For L-M algorithm has fewer iterations and is faster convergence than traditional BP algorithm, it is chosen as the training function of NN in this paper.

C. Reliability calculation method based on neural network

Reliability analysis methods introduced in [4], [5] and [6] can be divided into two categories, one is a method based on the gradient, and the other is Monte Carlo (MC) method. The general responses of engineering structures need to be solved by means of finite element method and structural responses are generally implicit. The limit state equation of structure can not be expressed explicitly and it is rather difficult to calculate the gradient of structural response. Therefore, the application of reliability analysis method based on gradient analysis to practical engineering is difficult. On the other hand, to the small probability events of actual engineering, direct MC simulation, needs a lot of analog sampling, so the calculation is inefficient. In order to improve the efficiency of MC simulation, similar functions replacing the surface of real limit state are searched, and then the number of finite element analog sampling is then reduced. This is the principle of Response Surface Method (RSM). But taking into account the issue of computational efficiency, quadratic polynomials are usually applied to approximate the limit state function in the response surface method. To complex limit state function, it will lead to a greater error. With its strong generalization ability, NN is able to simulate functions with arbitrary precision. It is a limit state function approximation method which is better than RSM.

In this paper, a neural network method of reliability calculation based on uniform design is suggested. This method is based on deterministic finite element analysis, NN and MC simulation. We will first generate training samples by deterministic finite element analysis, and then do regressions by the multi-layer forward networks. So the NN approximation of the real limit state function is obtained. On this basis, the failure probability of structural response is obtained by applying MC simulation to sample.

Different from usual method to generate training samples of neural network, they are generated by using uniform design method proposed in this paper. Compared with randomly generated sample of general random variable joint probability density function, the number of training samples generated by this method is fewer. They also have higher computational efficiency and accuracy. The steps of reliability calculation method based on uniform design and neural network are as follows:

1) Establish deterministic finite element model and determine the probabilistic model of the basic random variables which have effect on the structure;
2) Select training samples by uniform design method;
3) Put training samples into deterministic finite element model and obtain the responses of the structure which are concerned;
4) Establish a neural network with sufficient accuracy by using the samples and the responses resulted from finite element model;
5) Randomly generate sufficient random variables and make them go through the above neural network established in accordance with the distribution;
6) Obtain the reliability index of the structure.

D. Examples1

Suppose a limit state function in [7] is as follows

$$g(X) = \frac{1}{40}x_1^4 + 2x_2^2 + x_3 + 3 \qquad (3)$$

Where, $X = (x_1, x_2, x_3)$ are independent random variables with standard normal distributions. Consider $2f+1$ ($f = 1,2,3$) layers of each variable. That is, if $f = 2$, there is five layers. They are $\overline{X} \pm 2\sigma$, $\overline{X} \pm \sigma$ and $\overline{X}$. $\overline{X}$ is the mean and σ is the standard deviation.

RSM based on polynomials, MC and NN sampling with uniform design are used to calculate the failure probability of this problem. The results are shown in table Ⅰ.

TABLE I. RESULTS OF EXAMPLE 1

Methods		Failure Probability($P_f \times 10^{-4}$)	Numbers of Sampling
RSM	$f = 1$	3.2670	none
	$f = 2$	2.7614	none
	$f = 3$	2.2132	none
MC		3.12	3560850
NN		3.195	55

It can be seen that RSM based on polynomials is more sensitive to the parameter f . The result of NN which sampling with uniform design is more accurate and stable than that of RSM based on polynomials.

E. Examples2

Fig.2 is a calculation diagram of plane frame structure with three spans and twelve layers. Elasticity modulus of each element is $E = 2.0*10^7 Pa$. The relationship between inertia moment and area of element section is $I_i = \alpha_i A_i^2 (i = 1,2,3,4,5)$. The area A_i of element section and external load P are chosen as random variables. To consider the condition of normal use, the maximum allowable deformation $[U]$ is equal to 0.096 according to the requirements of the specification. Therefore, the limit state function can be expressed as:

$$G = 0.096 - U_{\max}(A_1, A_2, A_3, A_4, A_5, P) \ldots\ldots \ldots.(4)$$

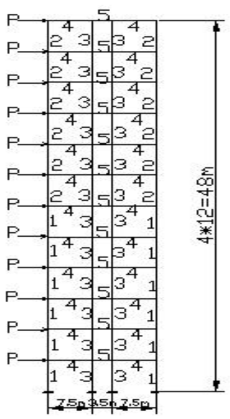

Figure 2. Framework diagram

The statistical characteristics of variables are shown in table Ⅱ.

TABLE II. STATISTICAL CHARACTERISTICS OF VARIABLES

Variables	Mean	Standard Deviation σ	Distribution Type	α_i
A_1 (m^2)	0.25	0.025	Log-normal	0.08333
A_2 (m^2)	0.16	0.016	Log-normal	0.08333
A_3 (m^2)	0.36	0.036	Log-normal	0.08333
A_4 (m^2)	0.2	0.02	Log-normal	0.26670
A_5 (m^2)	0.15	0.015	Log-normal	0.20000
P (KN)	30	7.5	Extreme-I	

To calculate the reliability of the frame structure, finite element model of ANSYS is established.

There are six variables. In accordance with the distribution, $\overline{X}$ and $\overline{X} \pm \sigma$, $\overline{X} \pm 2\sigma$ are selected to make up test combinations with six factors and five levels. In order to ensure the accuracy of response surface, uniform distribution experimental design table $\mathbf{U}_{25}(5^6)$ in [1] is used to decide 25 tests. Put these into deterministic finite element model established above and then the displacements of the structure are obtained.

Establish a NN with six inputs and one output by using the samples and the displacements obtained. According to the distribution of random variables, 5000 numbers are randomly generated. Then take the top 10 numbers of each random samples array as test inputs. The network training will begin after the samples are normalized. L-M algorithm is applied to networks training. Since the failure probability here is about 0.07, there are at least 1429 simulations as required by MC. After the completion of neural network training, MC simulation will only be carried out in the neural network approximation function based method proposed in this paper. Thus the computing efficiency is greatly improved.

5000 random selected samples will undergo MC simulation through this network. The result is 379 samples are beyond the limit 0.096. Therefore, the failure probability of this example is:

$$P_{f(KJ)} = \frac{379}{5000} = 0.0758 \qquad (5)$$

Reference [6] calculated the failure probability of this example using 2000 times of MC importance sampling simulation and general RSM. The results are shown in Table Ⅲ. We can see that method proposed in this paper has a fairly good accuracy and is more improved than the general RSM.

TABLE III. RESULTS OF THE EXAMPLE USING DIFFERENT METHODS

Methods	Failure Probability	Reliability Index
MC importance sampling	0.075058	1.4391
RSM	0.073	1.4538
NN which sampling with uniform design	0.0758	1.4339

III. Calculation of the Reliability of Concrete-Filled Steel Tubular Arch Bridge

Jiefang Bridge in Guangzhou shown in Fig. 3 is a big-span bridge crossing Zhujiang River. With the main bridge of 55m+83.6m+55m three-span continuous, it is a bottom-layer concrete-filled steel tubular tied arch without wind bracing.

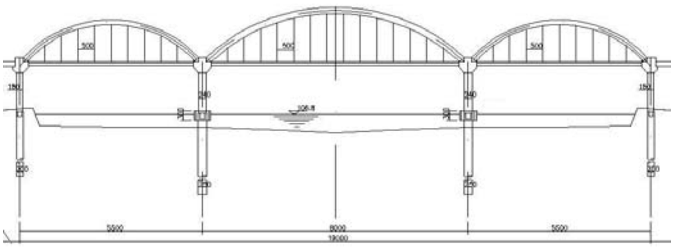

Figure 3. The bridge diagram (unit:m)

Full bridge model is established by ANSYS. Beam3 is used to simulate arch ribs, vertical and horizontal beams and bridge piers. Link10 simulated booms and tie members. And SHELL63 simulated deck. The model is shown in Fig. 4.

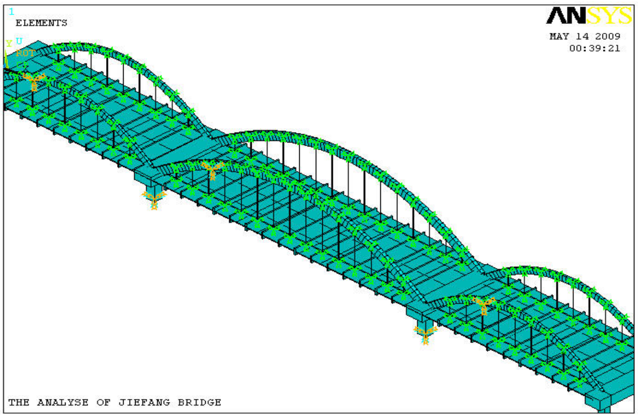

Figure 4. The bridge model established in ANSYS

In this paper, only the role of dead load and vehicle load has been considered during reliability analysis. The reliability of arch ribs is mainly analyzed.

The arch rib and vault, 1/4 arch ribs and arch foot are determined as controlling cross sections. According to [8], live load is arranged in the most unfavorable position of failure cross section.

By analyzing the full bridge model, main bearing limit states have been determined. When heavy vehicle of fleet in the mid-span, or 1/4 rib with maximum moment, and or foot arch with maximum axial force, bearing limit state is that the maximum vertical displacement of rib caused by cars and crowd is beyond the limit L/1000, or a cross section of arch ribs fails because of exceeding the pull or pressure limit.

Therefore, the limit state function of each arch rib can be expressed as follows:

$$G_{1i} = \sigma_{\lim p} - \sigma_{pi} \quad \text{……………………(6)}$$

$$G_{2i} = \sigma_{\lim l} - \sigma_{li} \quad \text{……………………(7)}$$

$$G_{3i} = u_{\lim} - u_i \quad \text{……………………(8)}$$

Where, $\sigma_{\lim p}$ is the maximum compressive stress allowed, σ_{pi} is the compressive stress of arch rib calculated, $\sigma_{\lim l}$ is the maximum tensile stress allowed, σ_{li} is the tensile stress calculated, $u_{\lim}$ is the maximum vertical displacement allowed and u_i is the maximum vertical displacement calculated. σ_{pi} , σ_{li} and u_i are results of heavy vehicle of fleet setting in section i ($i = 1,2,3,4,5,6$) , so there are 18 limit state functions.

Through analysis and comparison, 15 random variables which have considerable influence on the reliability index of arch ribs have been taken into account. Table Ⅳ lists the corresponding laws of probability and statistics of 15 variables.

TABLE IV. Statistical Characteristics of Variables

Variables	**Variable Name**	**Distribution Type**	**Mean**	**Coefficient of Variation**
E (Pa)	Elastic modulus of pipe of arch rib	Normal	2.1e11	0.1
E_1 (Pa)	Elastic modulus of concrete of arch rib	Normal	3.0e10	0.1
A_1 (m^2)	Area of pipe of arch rib of mid-span	Log-normal	0.9449	0.05
A_2 (m^2)	Area of concrete of arch rib of mid-span		1.5772	0.05
A_3 (m^2)	Area of pipe of arch rib of side-span		0.0687	0.05
A_4 (m^2)	Area of concrete of arch rib of side-span		0.8885	0.05
D (N/m^3)	Steel volume density	Normal	15127	0.05
D_1 (N/m^3)	Concrete volume density	Normal	14549	0.05
I_1 (m^4)	Moment of bending inertia of A_1	Log-normal	0.0096	0.05
I_2 (m^4)	Moment of bending inertia of A_2		0.0749	0.05
I_3 (m^4)	Moment of bending inertia of A_3		0.0039	0.05
I_4 (m^4)	Moment of bending inertia of A_4		0.0224	0.05
P_d (N)	Deck	Extreme-I	31814	0.1
P_c (N)	Crowd loads		3500	0.1
P_v (N)	Vehicle load		10000	0.1

In accordance with the characteristics of the table of uniform design, more factors can take into account of more levels. In order to improve the precision of calculation, for any variable randomly generated 10^7 values based on their characteristics of distribution, these values are within the scope of $\overline{X} \pm 3\sigma$ and do not exceed $\pm 5\sigma$. For each above random variable, 11 levels have been analyzed. The levels are $\overline{X} \pm 5\sigma$, $\overline{X} \pm 4\sigma$, $\overline{X} \pm 3\sigma$, $\overline{X} \pm 2\sigma$, $\overline{X} \pm \sigma$ and $\overline{X}$. Select the $\mathbf{U}_{110}\left(11^{15}\right)$ of uniform distribution design table to test design.

By the deterministic finite element model, the above 110 working conditions have been calculated, and critical data of study sections of each working condition obtained. Establish a MLP with the input layer of 15 units and the output layer of one unit. The L-M algorithm has been applied. After several attempts and checking the generalization ability of the network, the hidden layer is determined to be 29 units. The results obtained from deterministic finite element analysis is normalized and the neural network training is carried out, and then, 10 test samples among 10000 samples randomly generated are taken, the accuracy of neural network trained is checked. If the precision is sufficient, the reliability can be calculated. If it is not enough, network structure and the training function will be adjusted.

Corresponding to each condition, an independent neural network is established. After completion of the network training, 5000000 samples are randomly generated and applied to pseudo MC simulation to realize reliability calculation.

TABLE V. RIB FAILURE PROBABILITY OF EACH CONDITION

Failure Mode		Max	Min	Failure Probability
When heavy vehicle fleet is in the middle of mid span	MCSR(Pa)	-218.0e6	-81.3e6	0.0000014
	MSSR(Pa)	109.2e6	38.1e6	0
	MVDR(m)	-0.0467	-0.0181	0
When heavy vehicle fleet is in the 1/4 cross of mid span	MCSR(Pa)	-254.1e6	-78.1e6	0.0000738
	MSSR(Pa)	150.0e6	34.5e6	0
	MVDR(m)	-0.0598	-0.0144	0
When heavy vehicle fleet is in the arch foot of mid span	MCSR(Pa)	-176.3e6	-65.4e6	0
	MSSR(Pa)	88.3e6	33.8e6	0
	MVDR(m)	-0.0188	-0.0043	0
When heavy vehicle fleet is in the middle of side span	MCSR(Pa)	-250.9e6	-78.1e6	0.0001017
	MSSR(Pa)	143.6e6	49.5e6	0
	MVDR(m)	-0.0391	-0.0179	0
When heavy vehicle fleet is in the 1/4 cross of side span	MCSR(Pa)	-279.0e6	-102.5e6	0.00013
	MSSR(Pa)	193.1e6	64.3e6	0
	MVDR(m)	-0.0256	-0.0101	0
When heavy vehicle fleet is in the arch foot of side span	MCSR(Pa)	-201.8e6	-80.1e6	0
	MSSR(Pa)	106.7e6	37.9e6	0
	MVDR(m)	-0.0153	-0.0087	0

Table Ⅴ is the summary of failure probability of arch ribs in different working conditions. In the table, the displacement and stress are the results of equivalent treatment of the concrete of arch ribs as steel. Where, MCSR is present for the max compressive stress of rib, MSSR is present for the max tensile stress of rib and MVDR is present for the max vertical displacement of rib.

As far as the entire arch ribs system can be considered as tandem structure stated in [9], once a failure condition occurs as stated in the TableⅤ, the entire system can be seen as a failure. The tandem structure of rib system is shown in Fig. 5.

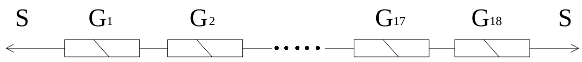

Figure 5. Tandem structure of rib system

The system failure probability can be generally estimated between 0.00013 and 0.0003269 according to interval estimation method. It seems from the statistical results of the failure that the probability that the maximum vertical displacement exceeds the normal use of limit state and the tensile stress of the upper edge exceeds the normal use limits is almost zero. The rib system is prone to pressure damage in the lower edge. Then 5000000 random samples are calculated, and the failure occurs 751 times. According to the Monte Carlo method for system reliability calculation, the failure probability is 0.0001502. Its corresponding reliability index is 3.6. Therefore, the reliability index of arch ribs of Jiefang Bridge is 3.6.

In general, the reliability index of arch ribs of Jiefang Bridge is relatively high and it can meet the requirement of the specifications.

IV. CONCLUTIONS

In this paper, the uniform design method is used in selecting neural network training samples and the method of calculating reliability based on uniform design and neural network have been brought forward. With Jiefang Bridge in Guangzhou as engineering background, the calculation of the reliability of arch ribs of concrete-filled steel tubular arch bridge is realized. The conclusions are as follows:

1) Uniform design method is an efficient method of experimental design. Since the uniform design has higher precision and generalization ability, the sampling of neural network training samples with many random variables can be more easily realized.

2) The calculation method of reliability based on uniform design and neural network has taken advantage of uniform design method which can find out the relationship between the relatively complex variables through fewer times of deterministic experiments. It combines the characteristics of neural network. Applying this method and carrying out deterministic finite element analysis with ANSYS, the reliability of arch bridge is achieved.

3) By means of solving the reliability index of Jiefang Bridge in Guangzhou, the reliability index of arch ribs of Jiefang Bridge is relatively high and the reliability of its given design can meet the requirement of the specifications. Moreover, the calculation method of reliability based on uniform design and neural network is proved that it can be used to solve the reliability of the actual structure component or the system. This is effective and practical calculation method with higher accuracy and reliability.

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