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On Nonparametric Estimation for the Frontier Elasticity of Output with DEA Model

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***Abstract*—the conventional aggregate production function model such as Cobb-Douglas requires strong assumptions such as the constant elasticity and the full efficiency of the technique. Without restrictive assumptions, DEA involves the use of linear programming methods to construct a non- parametric piecewise frontier over the data to estimate the efficiency. However, the DEA model is being short of production function form. In this paper, we combine the advantages of the DEA and the varying elasticity production function to estimate the Frontier elasticity: firstly, estimates the technical efficiency using the DEA model, then estimates the Frontier elasticity of output with varying elasticity production functions. It is showed that the Frontier elasticity is different from that of the conventional one.**

Keywords- efficiency; elasticity; production function; data envelope analysis; nonparametric estimation

I. INTRODUCTION

Production functions are a fundamental component of all economics. Production functions relate productive inputs (e.g. capital, labor) to outputs, reflect the effect and influence of production factor on output at certain technological conditions. Cobb-Douglas production function is preferred for its simple structure, meaningful parameter and easy estimation. Assuming two factors production, physical capital (K) and labor (L), the Cobb-Douglas production function is as follow:

$$Y = AK^{\alpha} L^{\beta} \quad (1)$$

However, the conventional economic growth model such as Cobb-Douglas production function requires unrealistically strong assumptions, such as the constant elasticity and the full efficiency of the technique.

Without restrictive assumptions, DEA involves the use of linear programming methods to construct a non-parametric piecewise frontier over the data. However, the DEA model is being short of the form of production function.

Iwata, Khan, and Murao[18], Bing Xu, Berlin Wu[3] applied a nonparametric method to estimate the varying elasticity of the capital and the labor. Ahmad [1], Bing Xu, Junzo Watada[4], Shangfeng Zhang, Bing Xu [16], Luo Xian-hua, Yang Zhen-hai, Zhou Yong [11] introduces the nonparametric varying-coefficients model to estimate varying output elasticity of capital and labor force.

In this paper, we combine the advantages of the DEA and the varying elasticity production function to estimate the Frontier elasticity: firstly, estimates the technical efficiency using the DEA model, then estimates the Frontier elasticity of output with varying elasticity production functions. The paper is organized as follows. In the following section contains a description of our data, and the estimation of efficiency with DEA model. The estimation and comparison of the frontier elasticity are given and discussed in Section III. The final section contains concluding remarks.

II. THE DATA AND THE EFFICIENCY ESTIMATION

A. description of data

The main variables contain Gross Domestic product (Y), Capital (K), Labour force (L), and Human Capital (H). We choose the 28 provinces of China at the year 2007. In order to eliminate the influence of inflation, we calculate the true data on the base year of 1990. Gross Domestic Product, which stands for output in the paper, is calculated by expenditure approach. The number of labour force is calculated by total employed persons at the year-end. In this paper, we follow Jun Zhang [15] to measure the capital, and follow Xiangjun Tang [19] to measure the human capital.

B. Introduction to DEA model

Modern efficiency measurement begins with Farrell [8] who drew upon the work of Debreu [7] and Koopmans [10] to define a simple measure of firm efficiency which could account for multiple inputs. Frontiers have been estimated using many different methods over the past 50 years. The two principal methods are: data envelopment analysis (DEA) and stochastic frontiers, which involve mathematical programming and econometric methods, respectively. DEA involves the use of linear programming methods to construct a non-parametric piecewise frontier over the data, so as to be able to calculate efficiencies relative to this surface.

TABLE I. THE EFFICIENCY AND FRONTIER GDP

Provinces	GDP Y	Efficiency *E*	Frontier GDP FY=Y/E	Education H
anhui	1069.41	0.68	1570.35	7.41
beijing	2198.93	0.41	5389.53	11.84
fujian	1984.40	0.65	3043.56	8.27
gansu	792.46	0.28	2810.14	7.01
guangdong	4369.04	0.80	5468.14	9.4
guangxi	1008.55	0.35	2889.83	8.45
guizhou	427.86	0.30	1440.61	7.05
hebei	3368.14	0.68	4924.18	8.59
henan	2326.59	0.46	5046.83	8.53
heilongjiang	1321.50	0.50	2627.24	9.14
hubei	1866.05	0.49	3800.51	8.54
hunan	1503.64	0.43	3464.61	8.61
jilin	1168.52	0.46	2551.35	9.05
jiangsu	5699.00	1.00	5699.00	8.7
jiangxi	1061.67	0.41	2608.53	8.72
liaoning	3823.44	1.00	3823.44	9.29
neimenggu	1285.55	0.36	3570.97	8.65
ningxia	226.12	1.00	226.12	8.12
qinghai	140.53	1.00	140.53	7.48
shandong	5616.87	0.99	5696.62	8.56
shanxi	1112.76	0.28	4017.18	9.22
Shan-xi	1106.36	0.31	3603.78	8.62
shanghai	5368.95	1.00	5368.95	11.41
sichuang	1842.65	0.35	5205.23	7.63
tianjin	2645.82	1.00	2645.82	10.46
xinjiang	623.23	0.33	1894.32	8.89
yunnan	769.74	1.00	769.74	6.92
zhejiang	3756.10	0.70	5350.57	8.5

Efficiency measurement has been a subject of tremendous interest as organizations have struggled to improve productivity. Reasons for this focus were best stated fifty years ago by Farrell [8] in his classic paper on the measurement of productive efficiency. Twenty years after Farrell's seminal work, and building on those ideas, Charnes et al. [5], responding to the need for satisfactory procedures to assess the relative efficiencies of multi-input multi-output production units, introduced a powerful methodology which has subsequently been titled data envelopment analysis (DEA). Since the advent of DEA in 1978, there has been an impressive growth both in theoretical developments and applications of the ideas to practical situations. Banker et al. [2] extended the earlier work of Charnes et al. [5] by providing for variable returns to scale (VRS). Wade D et al [18] provide a sketch of some of the major research thrusts in data envelopment analysis (DEA) over the past three decades.

The CRS assumption is only appropriate when all DMU's are operating at an optimal scale (i.e one corresponding to the flat portion of the LRAC curve). Imperfect competition, constraints on finance, etc. may cause a DMU to be not operating at optimal scale. Banker et al. [2] suggested an extension of the CRS DEA model to account for variable returns to scale (VRS) situations. The use of the CRS specification when not all DMU's are operating at the optimal scale will result in measures of TE which are confounded by *scale efficiencies* (SE). The use of the VRS specification will permit the calculation of TE devoid of these SE effects.

The VRS linear programming problem can be provide as:

$$\begin{aligned}&\text{Max}_{\theta,\lambda}\ \theta,\\ &\text{st}\ \ -y_i + Y\lambda \geq 0,\\ &\theta x_i - X\lambda \geq 0,\\ &N1'\lambda = 1\\ &\lambda \geq 0,\end{aligned} \tag{2}$$

where N1 is an N×1 vector of ones. This approach forms a convex hull of intersecting planes which envelope the data points more tightly than the CRS conical hull and thus provides technical efficiency scores which are greater than or equal to those obtained using the CRS model. The VRS specification has been the most commonly used specification in the 1990's.

Many studies have decomposed the TE scores obtained from a CRS DEA into two components, one due to scale inefficiency and one due to "pure" technical inefficiency. This may be done by conducting both a CRS and a VRS DEA upon the same data. If there is a difference in the two TE scores for a particular DMU, then this indicates that the DMU has scale inefficiency, and that the scale inefficiency can be calculated from the difference between the VRS TE score and the CRS TE score. Figure1 illustrate this theory.

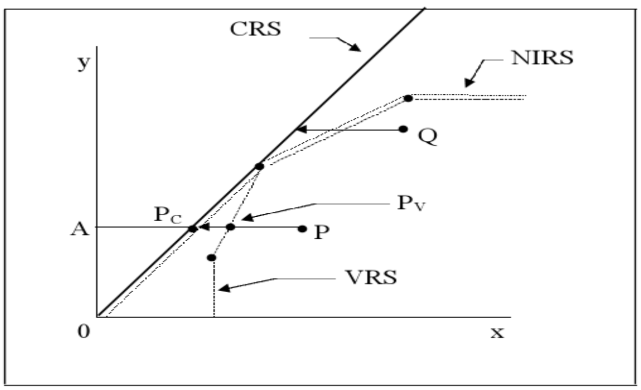

Figure 1. Calculation of scale economies in DEA

C. The results of Efficiency

In this paper, we use the BCC model to calculate the efficiency, with two inputs, capital and labor force, and one output GDP. The results of the efficiencies are estimated by computer program of DEAP version 2.1, and it is showed in table 1.

As can been seem from table 1: (1) Seven provinces, Jiangsu, liaoning, ningxia, qingha, shanghai, Tianjin, and Yunnan, are under the adequate efficiency 1, and one province Shandong has efficiency of 0.986 near to adequate efficiency. (2) Six provinces, Anhui, Fujian, guangdong, hebei, heilongjiang, and Zhejiang have a higher efficiency between 0.5 and 0.8, and the residual fifteen provinces have a lower efficiency under 0.5.

Will the provinces with higher human capital related to the higher efficiency? Figure 2 shows the relationship of human capital and efficiency. The upward regression line reveals the positive correlation of the two.

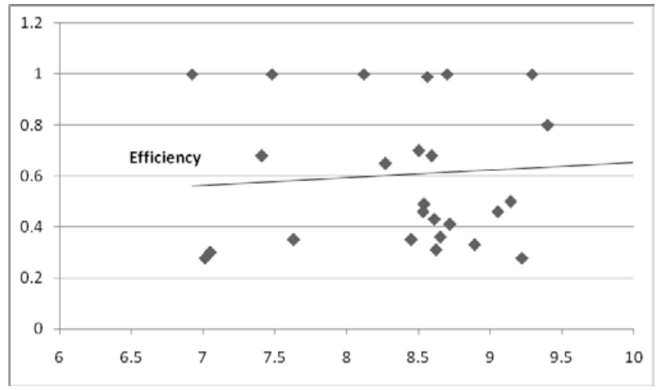

Figure 2. Human capital and efficiency

III. ESTIMATION OF THE FRONTIER ELASTICITY

A. Model specification and estimation methodology

Being a classical linear model, the parameters α and β are fixed for model (1). Varying coefficient models are a useful extension of classical linear models. They arise naturally when one wishes to examine how regression coefficients change over different groups characterized by certain covariates such as age. The potential of such a modeling technique got fully explored by the seminal work [Cleveland, Grosse and Shyu (1991), Hastie and Tibshirani (1993), Jianqing Fan, Wenyang Zhang (1999), J.Z. Huang, C.O.Wu, and L. Zhou (2002), and Jinhong You and Gemai Chen (2006) etc].

By combining the varying-coefficients model with C-D production function, we could construct a varying-time elasticity Production function model below, in which A is models as the function of time:

$$Y = e^{\lambda(H)} K^{\alpha(H)} L^{\beta(H)} \tag{3}$$

Varying-coefficient models, including generalized additive models as well as dynamic generalized linear models as special cases, are linear in the regresses but their coefficients are permitted to change smoothly as function of other variables. Ahmad [1] Bing Xu, Junzo Watada [4] Shangfeng Zhang, Bing Xu[16], Luo Xian-hua, Yang Zhen-hai, Zhou Yong[11] allowed the coefficients to change as the function of time. As the cross section data is used in this paper other than time series. Different to the former research, the coefficients are allowed to change as the function of human capital in model (3) in this paper.

We can deduce econometric model (4) by adding restriction α(H)+β(H)=1, taking logarithm and adding random item to model (3):

$$y = \lambda(H) + \alpha(H)k + \varepsilon \tag{4}$$

Here y=log(Y/L), and k=log(K/L). Obviously, model (4) is typical varying-coefficients model. One should note that in model (4) we have not restricted λ(H) and α(H) to have a fixed impact on the dependent variable, and assume that the functions λ(H) and α(H) possess about the same degrees of smoothness and hence they can be approximated equally well in the same interval.

For each given H_0, approximate the functions λ(H) and α(H) Locally as:

$$\lambda(H) \approx \lambda(H_0) + \lambda'(H_0)(H - H_0) \equiv \lambda_0 + \lambda_1 (H - H_0)$$
$$\alpha(H) \approx \alpha(H_0) + \alpha'(H_0)(H - H_0) \equiv \alpha_0 + \alpha_1 (H - H_0)$$

For H is in a neighborhood of H_0, this leads to the following local weighed least-squares problem:

$$Min \sum_{i=1}^{n} \{\ln y_i - (\gamma_0 + \gamma_1 (H - H_0)) - (\alpha_0 + \alpha_1 (H - H_0)) \ln k_i$$
$$-(\beta_0 + \beta_1 (H - H_0)) \ln L_i\}^2 W(\frac{H}{h})$$

We give more weight to contributions from observations very close to than to those coming from observations that are more distant. We choose the kernel function of Gaussian and choose Bandwidth h with the method of cross-validation in this paper.

The solution for model (4) is [Cleveland, Grosse and Shyu (1991)]:

$$\hat{\theta}(H_0) = (Z^T W Z)^{-1} Z^T W y \qquad (5)$$

Hereinto

$$\theta(H_0) = (\gamma_0, \alpha_0, \gamma_1, \alpha_1)^T;$$
$$y = (y_1, y_2, ..., y_n)^T;$$
$$Z = \begin{bmatrix} 1 & k_1 & H - H_0 & k_1 (H - H_0) \\ 1 & k_2 & H - H_0 & k_2 (H - H_0) \\ \vdots & \vdots & \vdots & \vdots \\ 1 & k_n & H - H_0 & k_n (H - H_0) \end{bmatrix}$$

B. Comparision of the frontier elasticity

Varying elasticity production function model allows the variety of the elasticity of output to vary over different human capital levels. Then, is there any difference between normal elasticity and frontier elasticity? In this paper, we consider two production functions: varying elasticity production function and frontier varying elasticity production function.

B1.Varying elasticity production function model

$$Y = e^{\lambda_1(H)} K^{\alpha_1(H)} L^{1-\alpha_1(H)} \qquad (6)$$

With productivity A_1:

$$A_1 = \frac{Y}{K^{\alpha_1(H)} L^{1-\alpha_1(H)}} \qquad (7)$$

B2.Frontier varying elasticity production function model

$$FY = e^{\lambda_2(H)} K^{\alpha_2(H)} L^{1-\alpha_2(H)}, FY = Y / E \qquad (8)$$

With productivity A_2:

$$A_2 = \frac{FY}{K^{\alpha_2(H)} L^{1-\alpha_2(H)}} \qquad (9)$$

The detail results for models (6)-(9) are displayed in table 2. The comparisons for the elasticity of capital and labor force, as well as the comparison of productivity according to the varying elasticity production function and the frontier varying elasticity production function are showed in figures 3-5.

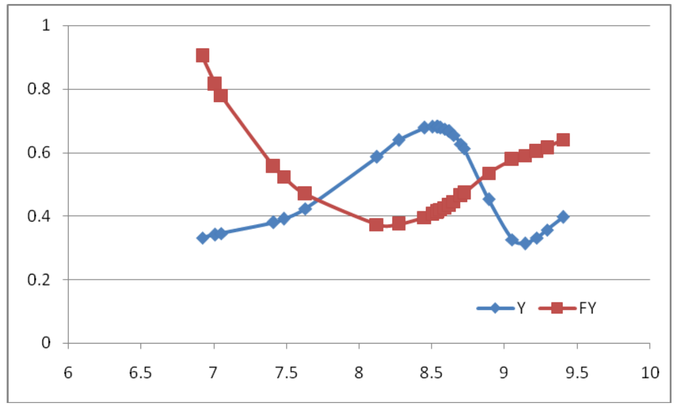

Figure 3. Comparisons for the elasicity of capital

As can be seen from the table 2 and figures 3-5, the results for the frontier varying elasticity production function are differently from that of the varying elasticity production function: (1) For the varying elasticity production function, as the rising of human capital, the elasticity of capital come through the inverted U-shaped pattern, and the elasticity of labour force come through the U-shaped pattern. But when it comes to the frontier varying elasticity production function, as the rising of human capital, the elasticity of capital come through the U-shaped pattern, and the elasticity of labour force come through the inverted U-shaped pattern. (2) the productivities are increasing with the increase in human capital for both of the two models, but the productivity for the frontier varying elasticity production function is higher than varying elasticity production function.

IV. CONCLUTIONS

The conventional aggregate production function model such as Cobb-Douglas requires strong assumptions such as the constant elasticity and the full efficiency of the technique. Without restrictive assumptions, DEA involves the use of linear programming methods to construct a non- parametric piecewise frontier over the data to estimate the efficiency. However, the DEA model is being short of production function form. In this paper, we combine the advantages of the DEA and the varying elasticity production function to estimate the Frontier elasticity: firstly, estimates the technical efficiency using the DEA model, then estimates the Frontier elasticity of output with varying elasticity production functions.

TABLE II. THE RESULTS

Provinces	Elasticity in Model (6)			Frontier Elasticity in Model (8)		
	Capital	Labor	Productivity	Capital	Labor	Productivity
anhui	0.380	0.620	0.386	0.555	0.445	0.762
beijing	0.639	0.361	0.472	0.374	0.626	0.933
fujian	0.342	0.658	0.365	0.815	0.185	0.906
gansu	0.397	0.603	0.832	0.641	0.359	1.106
guangdong	0.678	0.322	0.484	0.396	0.604	1.010
guangxi	0.346	0.654	0.365	0.779	0.221	0.879
guizhou	0.674	0.326	0.497	0.426	0.574	1.061
hebei	0.314	0.686	0.756	0.591	0.409	1.136
henan	0.681	0.319	0.490	0.412	0.588	1.042
heilongjiang	0.681	0.319	0.491	0.414	0.586	1.045
hubei	0.668	0.332	0.502	0.433	0.567	1.069
hunan	0.627	0.373	0.525	0.464	0.536	1.092
jilin	0.613	0.387	0.533	0.472	0.528	1.096
jiangsu	0.326	0.674	0.720	0.577	0.423	1.130
jiangxi	0.357	0.643	0.799	0.615	0.385	1.130
liaoning	0.654	0.346	0.510	0.446	0.554	1.080
neimenggu	0.586	0.414	0.459	0.374	0.626	0.873
ningxia	0.392	0.608	0.392	0.524	0.476	0.758
qinghai	0.678	0.322	0.494	0.420	0.580	1.054
shandong	0.332	0.668	0.781	0.602	0.398	1.136
shanxi	0.667	0.333	0.503	0.435	0.565	1.071
Shan-xi	0.424	0.576	0.406	0.470	0.530	0.763
shanghai	0.452	0.548	0.621	0.535	0.465	1.115
sichuang	0.332	0.668	0.366	0.904	0.096	0.980
tianjin	0.681	0.319	0.487	0.405	0.595	1.030
xinjiang	0.380	0.620	0.386	0.555	0.445	0.762
yunnan	0.639	0.361	0.472	0.374	0.626	0.933
zhejiang	0.342	0.658	0.365	0.815	0.185	0.906

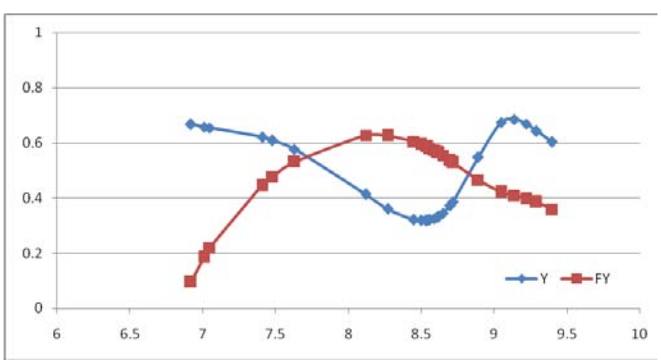

Figure 4. Comparisons for the elasicity of labor

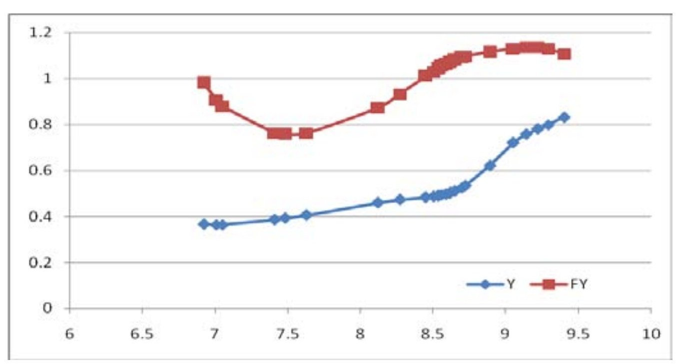

Figure 5. Human capital and the productivity

The empirical results find out the below conclusions: (1) Seven provinces, Jiangsu, liaoning, ningxia, qingha, shanghai, Tianjin, and Yunnan, are under the adequate efficiency 1, and one province Shandong has efficiency of 0.986 near to adequate efficiency, six provinces, Anhui, Fujian, guangdong, hebei, heilongjiang, and Zhejiang have a higher efficiency between 0.5 and 0.8, and the residual fifteen provinces have a lower efficiency under 0.5. (2) For the varying elasticity production function, as the rising of human capital, the elasticity of capital come through the inverted U-shaped pattern, and the elasticity of labour force come through the U-shaped pattern. But when it comes to the frontier varying elasticity production function, as the rising of human capital, the elasticity of capital come through the U-shaped pattern, and the elasticity of labour force come through the inverted U-shaped pattern. (3) the productivities are increasing with the increase in human capital for both of the two models, but the productivity for the frontier varying elasticity production function is higher than varying elasticity production function.

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