

Horizontal Input-Output Structure and Price Stickiness: Output Persistence and Inflation Inertia Under Monetary Shocks

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Abstract: The dynamic responses of output and inflation under monetary shocks are the emphasis of monetary economic cycle research. We introduce the empirical horizontal input-output structure into a standard new Keynesian sticky price macroeconomic model. We model the importance of intermediate input in production, and extend the research on the basis of this model. From China's input-output table, we estimate the ratio of intermediate good divided by gross output as 60%, which indicates the importance of intermediate good in the production and also the sophistication of production. From this ratio, we calibrate the elasticity of intermediate good to gross output (ϕ) is about 0.7. When introducing the empirical input-output structure, the output persistence and inflation inertia both increase significantly. The larger the ϕ is, the more the persistence is. This structure may explore a new transmission mechanism for monetary shocks. By applying both theoretical analysis and dynamic numerical simulation, we validate this persistence.

Keywords: Horizontal Input-Output Structure; Sticky Price; Output Persistence; Inflation Inertia; Money Supply Shock

Input-output table (is) an essential component in the description of price stickiness.

—Gordon(1990)

1 Introduction

Wassily•Leontief invented input-output model and analysis method in 1936, from then on, input-output table was widely composed and applied. Leontief got Nobel Prize by this contribution.

Input-output model describes the connections between different industries in an economies. Usually, the output of an industry is the input of other ones, and the input of an industry is composed by the output of other different industries. This kind of data characteristics is described in input-output matrices skillfully. Each column of this matrices gives all of the input of an industry and each row tells the distribution of the output of one industry using as other industries's input. Meanwhile extended input-output table reports value added and final uses. Besides the input, the output of one industry also contain the added value, such compensation of employees, depreciation of fixed assets, net taxes on production, operating surplus. Final uses tell the distribution of the output on final allocation, which contains final consumption, gross capital formation, and net export. input-output table plays an important role in national economic account.

The input-output table compiled by Bureau



of Economic Analysis(BEA) indicates that the interaction of industries share the properties of horizontal input-output structure and vertical input-output structure. Horizontal input-output structure means that the output itself is a very important input in one industry, while the vertical input-output structure means that the structure looks like a chain of production. For example, industry A's input is industry B's output, while A's output becomes C's input. Basu(1995) points out, usually the most important input of every industry is the output itself produces, which is equal to say that the number in diagonals are the largest. We compute the number of industries whose five largest input has its own output. In 17 industries, there is 15 in 2000, 14 in 2002, and 16 in 2005, which indicates that the feature that Basu(1995) emphasizes is significant in China's input-output table. As can be seen, horizontal input-output structure is significant in the economy. Taking the total output as a whole, intermediate input and added value compose the total output. The ratio of intermediate input to total output is an important index of input-output structure. Table 2 indicates that this ratio reaches or exceeds 50% in industrialized countries, from which point of view it is important in the economic activity.

From 1981, China starts to organize the input-output table. The pattern is organizing basic table every five years(when meet 2 and 7), and organizing extended table(when meet 0 and 5). Up to 2005, there are 10 tables, 1981, 1983,

1987, 1990, 1992, 1995, 1997, 2000, 2002 and 2005. Analyzing these input-output tables, we can study the variation of China's input-output tables. One significant feature is that the ratio of intermediate input to total output increases year by year and keeps at 60% these 10 years as is shown in Figure 1 which indicates the sophistication of production.

In the new Keynesian macro economic framework, the empirical significant horizontal input-output structure is more and more important. Basu(1995) introduces the intermediate input into firm's production function, and applies state dependent mechanism to depict technology's procyclicality. From Basu(1995) on, more and more literature introduce horizontal input-output structure to study business cycles. Bergin and Feenstra(2000) applies both trans-log demand structure and horizontal input-output structure, and find that improved sticky price model can explain the output persistence. Huang and Liu(2001) considers the case without capital, prove analytically that after introducing horizontal input-output structure, sticky price model can generate equivalent output persistence as sticky wage model. Huang, Liu and Phaneuf(HLP thereafter)(2004) introduces horizontal input-output structure into a new Keynesian model which contains both price rigidity and wage rigidity. The intermediate input ratio ϕ is different between inter WWII and post war which determines the different real wage cyclicality.

Standard sticky price model can not explain



the output persistence better (see Chari, Kehoe and McGrattan (2000)). The main reason is that under demand shocks firms increase output so that the marginal cost increase quickly and so the price increase which makes it difficult to generate output persistent. Follow this channel, many scholars improve the model to surpass the increment of marginal cost. Dotsey, King and Wolman (1997) considers variable capacity utilization. Erceg (1997) considers increasing return to scale and other factors.

Introducing horizontal input-output structure into standard sticky price model can significantly increase output persistence. The most important feature of horizontal input-output structure is final good can be used as both consumption and intermediate input. If there is price rigidity in intermediate input, under money supply shock the price of intermediate input change slower than capital and labor, which also lowers the degree of price change of capital and labor. So the price change of output becomes slower, the output becomes more persistent, and the inflation becomes more inertia. The empirical study in US supports the price rigidity of intermediate input. Carlton (1986) show that in industries such as steel, paper making, chemistry, stone and glass significant there exist significant price rigidity of intermediate input. Introducing intermediate input into production function supplies a new mechanism of monetary shocks. Horizontal input-output structure increases output persistence and inflation inertia.

After introducing horizontal input-output

structure, production function is

$$X_t = A_t \Gamma_t^\phi (K_t^\alpha L_t^{1-\alpha})^{1-\phi}$$

where parameter ϕ indicates the ratio of intermediate input to total cost, which is just as the output elasticity of intermediate input. HLP (2004) estimates ϕ is 0.7 in post war period, while we estimate China's ϕ also is 0.7 when applying the input-output data from 1992 to 2005. The larger ϕ is, the more persistent output is and the more inertia inflation is under money supply shock. When $\phi = 0.7$, the output elasticity of price is close to 1, it generate endogenous price rigidity, and impulse response function of output and inflation generate strong persistent.

This paper considers that the horizontal input-output structure plays an important role in the economic production. After introducing this factor into new Keynesian macro economic models, under money supply shock the output persistence and inflation inertia become strong significantly, and the persistence increases as the increase of ϕ . ϕ is equal to 0.7 in China. At this value, the model can generate quite strong endogenous price rigidity. Both theoretical analysis and numerical simulation validate these persistences.

The rest of the paper is organized as follows. Section 2 reviews the input-output structure and new Keynesian macroeconomics. Section 3 calibrates the key parameter ϕ in the model. Section 4 introduces empirical horizontal input-output structure into standard new Keynesian macroeconomic model, and construct



the model. Section 5 calibrates parameters. Section 6 first theoretically analyzes the mechanism of output persistence and inflation inertia under price rigidity and horizontal input-output structure, and then implement numerical simulation on output persistence and inflation inertia. Section 7 is the conclusion and extension.

2. Horizontal Input-Output Structure and New Keynesian Macro Economics Model

2.1 Horizontal Input-Output Structure

In China's statistical yearbooks, there are four tables related to the input-output structure, input-output basic flow table(intermediate use), input-output basic flow table(final use), input-output direct coefficient table and input output complete consumption coefficient table. We focus on the first table, input-output basic flow table(intermediate use), and the simplified case is in table 1. Basic flow table(final use) depicts the allocation of the industries's output at final consumption, gross capital formation, and net export. Direct consumption coefficients show the technology interaction of industries. The direct consumption a_{ij} means that , it takes industry j consuming a_{ij} units of industry i's output to produce one unit product, i.e., the ratio of industry j's consumption of industry i's output to industry j's total output. Complete consumption coefficient is industry j's direct and indirect consumption to industry i when industry j supplies one unit of final use.

There is plenty of literature research on China's input-output table. Here we mention some papers related to our paper. Shen and Wang(2006) defines the contribution coefficient of intermediate input, and show the relationship between the contribution coefficient and value added rate. They analyze the variation of this coefficient, and get the conclusion that the decrease of the coefficient of the new added intermediate input makes the decrease of the coefficient of the intermediate input and the value added rate. Liu and Cai(2008) applies yearly input-output data to analyze the trend variation of intermediate input from 1992 on. They apply the three sectors' direct consumption coefficient matrix and intermediate demand consumption matrix to analyze the influence of technology progress, industries structure change and price change at intermediate consumption. Cheng(2008) applies input-output approach and cross sectional data, and do comparative research on the development, the industry structure and the influence of productive service industry in China and 13 OECD countries. The research indicates that compared to OECD countries, the material input is large and service input is small in three sectors. In most OECD economies, nearly 70% of the productive service is put into service sector itself, while in China nearly 50% of the productive service is put into the second industry.

In the input-output table, intermediate input shares a large proportion. Table 2 shows the ratio of intermediate input to total output in main



countries. From this table we can see the ratio is at 40% to 50% in main industied countries, and the ratio in China is a little bit larger, 60%. Shen and Wang(2006) indicates that the high level of

ratio means the low value added rate. The high level of the ratio indicates the sophistication of production on one hand, and tells the low input-output efficiency.

Table 1 Input-output flow table (Intermediate Input) simplified paper

Input	Output	Department 1	...	Department N	Intermediate Input
Total Input(X)		$X_1 = \Gamma_1 + Y_1$...	$X_N = \Gamma_N + Y_N$	$\sum_{i=1}^N X_i$
Intermediate Input(Γ)		$\Gamma_1 = a_{11} + \dots + a_{1N}$...	$\Gamma_N = a_{N1} + \dots + a_{NN}$	$\sum_{i=1}^N \Gamma_i$
Department 1		a_{11}	...	a_{N1}	$\sum_{i=1}^N a_{i1}$
...	
Department N		a_{1N}	...	a_{NN}	$\sum_{i=1}^N a_{iN}$
Value Added(Y)		Y_1	...	Y_N	$\sum_{i=1}^N Y_i$
Fixed Capital Depreciation(δK)		δK_1	...	δK_N	$\sum_{i=1}^N \delta K_i$
Employee Compensation(WL)		WL_1	...	WL_N	$\sum_{i=1}^N WL_i$
Net Production Tax(T)		T_1	...	T_N	$\sum_{i=1}^N T_i$
Operational Profit(Π)		Π_1	...	Π_N	$\sum_{i=1}^N \Pi_i$

Table 2 The ratio of intermediate input to total output in main countries

Countries	China	USA	Russia	Japan	Germany	UK	Australia
Year	2000	1996	1995	2000	2000	2002	1996
Vaule Added Rate	0.3586	0.5666	0.5528	0.5418	0.4995	0.4658	0.4759
Γ / X	0.6414	0.4334	0.4772	0.4582	0.5005	0.5342	0.5240

Note: Calculate according to Shen and Wang(2006) , table 2. $\frac{\Gamma}{X} = 1 - R$, R is the value added rate.



International comparison shows that the ratio of intermediate input to total output is close to 50% in main countries, which indicates the importance of horizontal input-output structure in these countries. In recent 20 years, China's horizontal input-output structure increases significantly. Figure 1 depicts the variation of the ratio of intermediate input to total output from 1981 to 2005, where the sample points are 1981, 1983, 1987, 1990, 1992, 1995, 1997, 2000, 2002 and 2005. From this figure, the ratio increases year by year which indicates that China's economy becomes more and sophisticated as the opening and reform strategy.

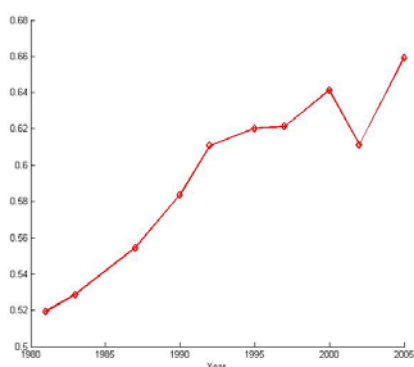


Figure 1 Ratio of intermediate input to total output

Note: The point of 1981, 1983, 1987 and 1990 is got according to Shen and Wang (2006), table 3.

2.2 New Keynesian Macro Economic Models

The new Keynesian macroeconomic models are a kind of models which introduces money, monopolistic competition, nominal rigidity into real business cycle models. It is a Keynesian with micro foundations. The review papers of new Keynesian model are Goodfriend and King(1997), Clarida, Gali and Gertler(1999), etc.

Nominal rigidity includes price rigidity and wage rigidity. At the time dependent framework, there are Calvo style and Taylor style, see Calvo(1983) and Taylor(1980). Chari, Kehoe and McGrattan(2000) finds that price rigidity alone can not generate enough output persistence, while Huang and Liu(2002) finds that sticky wage model can generate quite strong output persistence, which means a more driving monetary policy transmission. In these models the production function is standard Cobb-Douglas function with two factors capital and labor, while in part of literature there is intermediate input in the production function and this kind of Cobb-Douglas production function consist of intermediate input and the combination of labor and capital. Intermediate input in the production function fits the circular and round about production process.

Basu(1995) introduces intermediate input into production function. In his model, there is no capital, so intermediate input and labor form the Cobb-Douglas function. He depicts price rigidity in a state dependent style not the time dependent style. Bergin and Feenstra(2000) applies both trans-log demand structure and horizontal input-output structure, and find that the improved sticky price model can explain the persistence effect. Huang and Liu(2001) considers a model without capital, proves analytically that after introducing horizontal input-output structure, sticky price model can generate the same degree as sticky wage model.

Our paper calibrates a new Keynesian



macroeconomic model introducing horizontal input-output structure with China's data, and estimates the ratio of intermediate input to total output ϕ from China's input-output table.

3 Estimation of Key Parameters

In this part we apply two ways to calibrate the key parameters: ϕ , the ratio of intermediate input to total output.

3.1 Estimate θ_p , ϕ and α simutaneously

we can estimate three important parameters: substitution elasticity across output (θ_p), intermediate input' output elasticity ϕ and capital's output elasticity (α) through four equations at steady state.

The first equation is income distribution equality:

$$X = \Gamma + T + ZK + \Pi + WL \quad (1)$$

X is gross output, T is tax, Γ is intermediate input, Z is nominal rental cost, K is capital, Π is profit, WL is wage, and then

$$X - T = \Gamma + ZK + \Pi + WL$$

Since there is no tax in our theoretical model, $X - T$ is just X.

The second equation is wage distribution equality, which is derived from firms' labor demand:

$$\frac{WL}{X - T} = MC(1 - \phi)(1 - \alpha) \quad (2)$$

The third equation is capital distribution equality, which is derived from firms' capital demand:

$$\frac{ZK}{X - T} = MC(1 - \phi)\alpha \quad (3)$$

The fourth equation is intermediate input

distribution equality, which is derived from firms' intermediate input demand:

$$\frac{\Gamma}{X - T} = MC\phi \quad (4)$$

Sum equation (2) and (3),

$$\frac{WL + ZK}{X - T} = MC(1 - \phi) \quad (5)$$

Devide equation (4) by (5) and calculate,

$$\phi = \frac{\Gamma}{\Gamma + WL + ZK} \quad (6)$$

Substitute into equation (4) and calculate,

$$MC = \frac{\Gamma}{X - T} / \phi \quad (7)$$

Then

$$\theta_p = \frac{1}{1 - MC} \quad (8)$$

Substitute equation (7) into (3) and calculate

$$\alpha = \frac{ZK}{X - T} \frac{1}{MC(1 - \phi)} \quad (9)$$

Then from equation (6), (7), (8) and (9), we can get ϕ , MC, θ_p and α , and see table 3.

We find ϕ is about 70%, Γ / X about 60% in table 3, which indicates the ratio of intermediate input to total output is stable in this 15 years, and intermediate input is the most important input factor. Real marginal cost is about 0.9, which means the ratio of profit to total output is about 10%. By this data the substitution elasticity across intermediate good (θ_p) is about 11. The value of ϕ and θ_p is similar to US case. Basu and Fernald (1997, 2000) show, price cost markup is 1.12 when not considering capital utilization from which θ_p is about 11. HLP (2004) calculate $\phi = \Gamma / X$ as 0.68 from US input-output table 1997. In research of China's new Keynesian mode, Zhang (2008) gets θ_p as 4.61 from estimating structural equation,



which is lower than US case and our estimation. α is about 0.23, which is lower than many studies' result. He, Zhang and Shek(2007) estimates α as 0.6, and it is 0.33 in US case. The downward biased α can be explained as follows. Capital gain is represented by fixed capital depreciation(δK) in input-output table, by capital rental gain(ZK). Z is basically greater than δ , which means that part of profit Π belongs to capital gain.

3.2 Fix α and (or) θ_p , estimate ϕ

Fix θ_p equal to 4.61, α equal to 0.6, we estimate ϕ . From equation (9) and equation (8),

we can get:

$$\phi = 1 - \frac{ZK}{X - T} \frac{\theta_p}{(\theta_p - 1)\alpha} \quad (10)$$

Table 4 gives the ϕ value.

Also we fix α , and estimate both θ_p and ϕ . Substitute equation(7) into equation(9):

$$\phi = \frac{\alpha}{\alpha + ZK / \Gamma} \quad (11)$$

Meanwhile, we get MC from equation (7),

and then get θ_p from equation (8). Table

5 gives θ_p and ϕ .

Table 3 The calibration of MC, θ_p , ϕ and α

	1992	1995	1997	2000	2002	2005	平均值
MC	0.8806	0.8914	0.9282	0.9410	0.9098	0.8942	0.9075
θ_p	8.3785	9.2110	13.9344	16.9432	11.0826	9.4502	11.5000
α	0.2269	0.2140	0.1989	0.2264	0.2412	0.2649	0.2287
ϕ	0.7284	0.7323	0.7054	0.7191	0.7115	0.7732	0.7283
Γ / X	0.6108	0.6202	0.6212	0.6414	0.6112	0.6593	0.6273

Note: Calculate according to input-output basic flow table(intermediate input) 1992,1995,1997,2000,2002,2005.

Table 4 Fix α and θ_p , estimate ϕ

	1992	1995	1997	2000	2002	2005	平均值
ϕ	0.8845	0.8913	0.8842	0.8727	0.8652	0.8857	0.8806

Table 5 Fix α , estimate ϕ and θ_p

	1992	1995	1997	2000	2002	2005	平均值
ϕ	0.8764	0.8847	0.8784	0.8716	0.8598	0.8854	0.8806
θ_p	3.7305	3.8159	3.9277	4.4721	4.0454	4.5645	4.0927



Comparing between table 4 and table 5, we find ϕ significantly larger after fixed α as 0.6. According to the above two estimation, we choose the benchmark parameters as $\alpha = 0.3$, $\theta_p = 10, \phi = 0.7$ (We revise the result from 3.1 by increasing α and decreasing θ_p), and do sensitivity analysis in numerical experiment to check the robustness.

4 Model

This part shows the model. The model consists of three modules, households, firms and government. Government is the monetary authority.

4.1 Households

There is a plenty of households in the economy, normalized in a continuum [0,1]. Representative agent i maximizes its utility (Subscript i is omitted to simplify the notation without confusion):

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\sigma} C_t^{*1-\sigma} - \frac{1}{1+\eta} L_t^{1+\eta} \right\}$$

Where $C_t^* \equiv [bC_t^\nu + (1-b)\left(\frac{M_t}{P_t}\right)^\nu]^\frac{1}{\nu}$.

The budget constraint is:

$$P_t C_t + P_t I_t \left[1 + \phi \left(\frac{I_t}{K_{t-1}} \right) \right] + \frac{1}{R_t} B_t + M_t \quad (12)$$

$$\leq W_t L_t + Z_t K_{t-1} + \Pi_t + B_{t-1} + M_{t-1} + TR_t$$

The corresponding Lagrangian multiplier is λ_t . The subscript of K_{t-1} means K_{t-1} is the state variable at the end of $t-1$. $\phi \left(\frac{I_t}{K_{t-1}} \right) = \frac{\Psi}{2} \left(\frac{I_t}{K_{t-1}} \right)^2$ is the capital adjustment cost, where Ψ is the adjustment cost parameter.

The detailed adjusting approach is reported in Appendix which is available from the author upon request.

The law of motion of capital is:

$$I_t = K_t - (1-\delta)K_{t-1} \quad (13)$$

the corresponding Lagrangian multiplier is ξ_t .

Households' best choice of investment (I_t),

capital (K_t), consumption (C_t), labor (L_t),

money (M_t) and bond (B_t):

The first order condition for C_t is:

$$b[bC_t^\nu + (1-b)\left(\frac{M_t}{P_t}\right)^\nu]^\frac{1-\sigma}{\nu} C_t^{\nu-1} = \lambda_t P_t$$

The FOC for I_t :

$$\lambda_t \left[1 + \frac{3}{2} \Psi \left(\frac{I_t}{K_{t-1}} \right)^2 \right] = \xi_t$$

The FOC for K_t :

$$\xi_t P_t = +\beta E_t \{ \lambda_{t+1} (Z_{t+1} + P_{t+1} \Psi \left(\frac{I_{t+1}}{K_t} \right)^3) + \xi_{t+1} P_{t+1} (1-\delta) \}$$

The FOC for M_t :

$$(1-b)[bC_t^\nu + (1-b)\left(\frac{M_t}{P_t}\right)^\nu]^\frac{1-\sigma}{\nu} \left(\frac{M_t}{P_t}\right)^{\nu-1} \frac{1}{P_t} = \lambda_t - \beta E_t \lambda_{t+1}$$

The FOC for B_t : $\frac{\lambda_t}{R_t} = \beta E_t \lambda_{t+1}$

The FOC for L_t : $L_t^\eta = \lambda_t W_t$

4.2 Firms

There is a continuity of firms in the economy which is normalized in [0,1]. The representative firm is j . Subscript j is omitted to simplify the



notation as soon as there is no confusion.

Firms supply product that have some substitution with degree θ_p . Suppose there is a continuity of goods aggregators whose tasks are summing up discrepant good into final good X_t . These aggregators are competitors, and the price index can be derived from their optimal behaviors:

$$P_t = \left[\int_0^1 P_{jt}^{1-\theta_p} dj \right]^{\frac{1}{1-\theta_p}}$$

and firm j faces its demand curve:

$$X_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta_p} X_t.$$

In sticky price model, a firm adjusts its price in Calvo style, which means in every period firms can not adjust the price with probability α_p , and can do with probability $1 - \alpha_p$ accordingly.

Firm's optimal price P^* is:

$$P_t^* = \frac{\theta_p}{\theta_p - 1} \frac{E_t \sum_{i=0}^{\infty} (\beta \alpha_p)^i \Lambda_{t,t+i} \left(\frac{1}{P_{t+i}} \right)^{1-\theta_p} X_{t+i} MC_{t+i}^n}{E_t \sum_{i=0}^{\infty} (\beta \alpha_p)^i \Lambda_{t,t+i} \left(\frac{1}{P_{t+i}} \right)^{1-\theta_p} X_{t+i}} \quad (14)$$

Where $\Lambda_{t,t+i} = \frac{U_{C,t+i}}{U_{C,t}}$, MC_t^n is nominal

marginal cost, which is equal to the product of MC_t and price. MC_t can be derived from firm's cost minimization.

Firm's cost minimization problem is

$$\max_{L_t, K_{t-1}, \Gamma_t} - \left(\frac{W_t}{P_t} L_t + z_t K_{t-1} + \Gamma_t \right) + MC_t (A_t \Gamma_t^\phi [K_{t-1}^\alpha L_t^{1-\alpha}]^{1-\phi} - X_t)$$

$X_t = A_t \Gamma_t^\phi [K_{t-1}^\alpha L_t^{1-\alpha}]^{1-\phi}$ is firm's production

function, where the value added parts intermediate input Γ_t , capital K_{t-1} , and labor L_t compose

Cobb-Douglas production function form. Γ_t 's

output elasticity is ϕ , K_{t-1} 's is $\alpha(1-\phi)$ and

$$L_t \text{'s is } (1-\alpha)(1-\phi).$$

From the FOC for L_t we can get firm's labor demand function:

$$L_t = MC_t (1-\phi)(1-\alpha) \frac{X_t}{W_t} P_t.$$

From the FOC for K_{t-1} we can get firm's capital demand function:

$$K_{t-1} = MC_t \alpha (1-\phi) \frac{X_t}{Z_t} P_t.$$

From the FOC for Γ_t we can get firm's intermediate input demand function:

$$\Gamma_t = MC_t \phi X_t.$$

Z_t is the nominal capital rental cost.

Substitute the above three equations into production function, and get MC_t and MC_t^n .

$$MC_t = \frac{1}{A_t} \bar{\phi} \frac{(Z_t^\alpha W_t^{1-\alpha})^{1-\phi}}{P_t^{1-\phi}} \quad (15)$$

$$MC_t^n = \frac{1}{A_t} \bar{\phi} P_t^\phi (Z_t^\alpha W_t^{1-\alpha})^{1-\phi} \quad (16)$$

Where $\bar{\phi}$ is a constant constructed by α

and ϕ . From equation (16) we find the relationship among nominal marginal cost, intermediate input's price and other factors' prices.



When $\phi = 0$ intermediate input has no use in production, so its demand is 0 in equilibrium and the model becomes the standard sticky price model without intermediate input.

4.3 Money Supply Shock

Given the exogenous money supply, we suppose the growth rate of money is an AR 1 process:

$$\ln g_t = (1 - \rho_m) \ln g + \rho_m \ln g_{t-1} + \epsilon_{mt}$$

Where $g_t = \ln M_t - \ln M_{t-1}$, g is the steady state of money growth rate, which is set to 1 in the model. $0 < \rho_m < 1$, ϵ_{mt} is a normal distribution with zero mean and finite variance. In our impulse response experiments, we choose the initial value of the shock ϵ_0 so as the money stock increases by 1 percent after one year.

4.4 Equilibrium System

The equilibrium of the economic system is defined as follows:

Representative agent i chooses C_{it} , K_{it} , B_{it} , M_{it} and W_{it} , representative agent j chooses Γ_{jt} , $K_{j,t-1}$, L_{jt} , P_{jt} , and kinds of price: R_t , P_t , Z_t and W_t satisfies the following conditions:

1. Given wages and other firms's prices, firm chooses the quantities of factors and prices to solve its profit maximization problem;
2. Given prices and other households' wages, household chooses the quantities of consumptions, labors, money, bond, investment and wage to solve its utility maximization problem;
3. All the market(money, bond, capital, aggregate labor, aggregate output, net output) is clearing;

4. Money supply is an exogenous shock process (This paper does not mention to technology shock, so $A_t = 1$).

Market clearing condition:

1. Money market clearing: $M_t = M_{t-1} + TR_t$
2. Aggregate market clearing(gross resource constraint): $X_t = Y_t + \Gamma_t$
3. Bond market clearing: $B_t = 0$
4. Net output market clearing (net resource constraint):

$$C_t + I_t \left[1 + \frac{\psi}{2} \left(\frac{I_t}{K_{t-1}} \right)^2 \right] = Y_t$$

5 Calibration

Some parameters are from other research work while others are calculated by authors. In utility function, the relative risk aversion coefficient of consumption σ is 1; According to He, Zhang and Shek(2007), the relative risk aversion coefficient of labor η is 6.16. We estimate China's money demand function, and get $b = 0.10, \nu = -0.77$, where b is the relative weight of consumption between consumption and real money balance. The value of b indicates consumers pay far more attention to real money balance than consumption, which is consistent to reality of China. In production function, we estimate α as 0.3. In the aggregate production, we estimate θ_p as 10. In capital accumulation function, according to He, Zhang and Shek(2007), the depreciation rate in a quarter is 0.04. From 1989 to 2008, one year loan rate is 7.68% on average, and so quarter interest rate is



1.94%. According the relationship between interest rate and discount rate in steady state $R\beta = 1$, we can get the discount factor as $\beta = 0.98$. The capital adjustment cost parameter Ψ is adjusted according to the model, and the criterion is making the ratio of investment and output variance in the model is equal to it in the data. We estimate the process of the growth rate of

M2, ρ_m is equal to 0.59. $\alpha_p = 0.5$, which means on average a firm adjusts its price every half year. In literature α_p is set to 0.5 or 0.75. Larger the value mean larger price exogenous rigidity. Since we mainly consider the endogenous price rigidity by introducing intermediate input, set lower α_p is proper. The detailed calibration is seen in Appendix. Parameters are in table 6.

Table 6 Model Benchmark Parameters

Preference: $\frac{1}{1-\sigma} C^{*1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}$	$\sigma = 1, \eta = 6.16$
$C^* = [bC^\nu + (1-b)(\frac{M}{P})^\nu]^\frac{1}{\nu}$	$b = 0.10, \nu = -0.77$
Technology: $X = A\Gamma^\phi (K^\alpha N^{1-\alpha})^{1-\phi}$	$\alpha = 0.3, \phi = 0.7$
Aggregate Output: $X = (\int X_j^\frac{\theta_p-1}{\theta_p} dj)^\frac{\theta_p}{\theta_p-1}$	$\theta_p = 10$
Capital Accumulation: $K_{t+1} = I_t + (1-\delta)K_t$	$\delta = 0.04$
$\phi(\frac{I_t}{K_t}) = \frac{\Psi}{2} (\frac{I_t}{K_t})^2$	Ψ adgisted according to the model
Money growth rate: $\ln g_t = \rho_m \ln g_{t-1} + \epsilon_{mt}$	$\rho_m = 0.59$
Subjective Discount Factor	$\beta = 0.98$
Price Adjusted Probability	$1 - \alpha_p = 0.5$

6 Dynamic Analysis

6.1 Transimisson Mechnism Analysis

First of all, we analyze the mechanism of output persistence and inflation inertia under sticky price and horizontal input-output structure similar to Huang and Liu(2001).

Log linearize equation (14) around steady state:

$$\hat{P}_t^* = \beta \alpha_p E_t P_{t+1}^* + (1 - \beta \alpha_p) (\widehat{MC}_t + P_t) \quad (17)$$

Where $\hat{\cdot}$ means log linearizing variable around its steady state.

Log linearize equation (15):



$$\widehat{MC}_t = \alpha(1-\phi)(\widehat{\widehat{Z}}_t - P_t) + (1-\alpha)(1-\phi)(W_t - P_t)$$

substitute it into equation (17)

$$\widehat{P}_t^* = \beta\alpha_p E_t P_{t+1}^* + (1-\beta\alpha_p)(\alpha(1-\phi)Z_t + (1-\alpha)(1-\phi)\widehat{W}_t + \phi P_t) \quad (18)$$

The optimal price \widehat{P}_t^* is the weighted sum of the expectation of next period price and current period nominal marginal cost. In order to do further study, we abstract capital, $\alpha = 0$, which is common when talking about theoretical mechanism. Also suppose money is no utility ($b = 1$), and add a static money demand

function, $Y_t = \frac{M_t}{P_t}$, which does not change the

channel of the mechanism. Log linearize this equation we get $\widehat{Y}_t = M_t - P_t$.

Log linearize the FOC for C_t ,

$$\widehat{\lambda}_t = -\sigma C_t - \widehat{P}_t \quad (19)$$

Log linearize the FOC for L_t ,

$$\widehat{W}_t = \eta L_t - \widehat{\lambda}_t \quad (20)$$

Substitute equation (19) into equation (20),

$\widehat{W}_t = \eta L_t + \sigma \widehat{C}_t + P_t$. After abstracting capital,

there is $\widehat{Y}_t = \widehat{C}_t$, so $\widehat{W}_t = \eta L_t + \sigma Y_t + P_t$,

substitute this equation into equation (18)

$$\widehat{P}_t^* = \beta\alpha_p E_t P_{t+1}^* + (1-\beta\alpha_p)((1-\phi)(\eta L_t + \sigma Y_t) + P_t) \quad (21)$$

Use intermediate input's demand function, production function and gross resource constraint function, $\widehat{W}_t - P_t = \eta L_t + \sigma Y_t$, and $\widehat{L}_t = Y_t$, substitute into equation (21), get

$$\widehat{P}_t^* = \beta\alpha_p E_t P_{t+1}^* + (1-\beta\alpha_p)(\gamma_p Y_t + P_t) \quad (22)$$

Transform equation (22):

$$\widehat{P}_t^* - \widehat{P}_t = \beta\alpha_p (E_t P_{t+1}^* - \widehat{P}_t) + (1-\beta\alpha_p)\gamma_p Y_t \quad (23)$$

Where

$$\gamma_p = \frac{(\sigma + \eta)(1-\phi)}{1 + f(\phi, \theta_p)\eta} = (\sigma + \eta) \frac{(1-\phi)}{1 + f(\phi, \theta_p)\eta}$$

$$f(\phi, \theta_p) = \frac{\phi}{\phi + \theta_p(1-\phi)}$$

Combine equation (23) and money demand function, under money supply shock, total demand and output increase because of price rigidity. The optimal price will increase when total demand increases. The more sufficient the price adjustment is which is consistent with a larger price output elasticity, the less persistent the output is, vice versa. This elasticity is γ_p , and endogenous price rigidity is negative correlated to γ_p . Smaller γ_p is, larger price rigidity is. If $\gamma_p < 1$, model can generate endogenous persistence. When $\phi = 0$ (no intermediate input), $\gamma_p = \sigma + \eta$, under benchmark calibration, $\gamma_p = 7.16 > 1$, so model can not generate endogenous price rigidity. This is the reason why sticky price model can not generate endogenous price stickiness. When $\phi > 0$, let

$$g = \frac{(1-\phi)}{1 + f(\phi, \theta_p)\eta}, \quad \frac{\partial f(\phi, \theta_p)}{\partial \phi} > 0$$

means $\frac{\partial g}{\partial \phi} < 0$, which means when $\phi = 0$, g reaches its maximum, 1. When $\phi = 0.7$, $g = 0.1385$, $\gamma_p = 0.9920 < 1$, so the model generate endogenous output persistence. Figure2



shows the picture of γ_p as a function of ϕ given θ_p and σ . γ_p is a decreasing function of ϕ . γ_p approximate 1 when $\phi = 0.7$.

The larger ϕ is, the smaller the reaction of price to output variation. Inflation also will be more inertia when output is more persistent. When $\alpha \neq 0$, Z_t appears in the expression of marginal cost. Because the adjustment of Z_t is sufficient, introducing capital will weaken the output persistence, that is introducing capital is detrimental.

6.2 Numerical Simulation

We log linearize the nonlinear system around steady state, get the linear equilibrium system, and, implement money supply shock at this system, and get impulse response function of all the variables. The strength of money supply shock is making the money stock increases by 1 percent after one year.

Figure 3 shows the impulse response function of main variables under money supply shock.

The variation of ϕ generates significant effect on the dynamics of output and inflation. Accurately speaking, the larger the ϕ is, the more persistent the output is and inertia the inflation is. In order to analyze it, we set ϕ as 0, 0.2, 0.5, 0.7 and 0.9, and then implement impulse response function. Figure 4 and 5 show the impulse response function of output and inflation under money supply shock.

From figure 4, as ϕ increases, the intensity and persistence of output get great increasing. We

analyze the reaction intensity from the reaction intensity at the first period. When $\phi = 0$, i.e., intermediate input does not play a role in production. Clearly in equilibrium $\Gamma = 0$, which is consistent with standard sticky price model. The reaction of output is less than 0.2%. When $\phi = 0.5$, the reaction of output exceeds 0.4%. When $\phi = 0.7$, the reaction is close to 0.7%. When $\phi = 0.9$, i.e., the relative importance of intermediate input is 90%, the reaction of output is close to 1%. From the perspective of output persistence, we can see the output persistence is increasing in the figure. At the same time, we applies contract multiplier to represent persistence, which means the ratio of reaction at the contract expiration to the first period. In our benchmark setting $\alpha_p = 0.5$, so the average time that firms keep price unchanged is $1/(1-\alpha_p) = 2$, i.e., 2 quarters(half year). We choose the ratio of reaction at period 3 to period 1, and compute the contract multiplier. Table 7 gives the results. As ϕ increases, contract multiplier increases. When $\phi = 0.7$, the contract multiplier increases significantly. Especially when ϕ is 0.9, contract multiplier increase to 0.5.

From figure 5 we can as ϕ increases the reaction of inflation at the initial period is repressed which makes the reaction of output is reinforced and also the inflation inertia is reinforced. We apply contract multiplier to represent the inflation inertia. Table 8 show that as ϕ increases, contract multiplier increases. When $\phi = 0.5$ contract multiplier increases significantly. When $\phi = 0.7$, contract multiplier



exceeds 0.15. Especially ϕ is 0.9, contract multiplier is close to 0.5.

6.3 Sensitivity Analysis

Since there is a little different from our calibration and literature when calibrating α and θ_p , we do sensitivity analysis to α and θ_p . Figure 6 shows the sensitivity analysis to α . We find that when α is 0.3 or 0.6, the difference of output reaction between these two value is small, which indicates that the result is robust when α is in this range. Figure 7 shows the sensitivity analysis to θ_p . We find that when θ_p is 5 or 10, the diffencen of output reaction between these two value is small, which indicate that the result is robust when θ_p is in this range

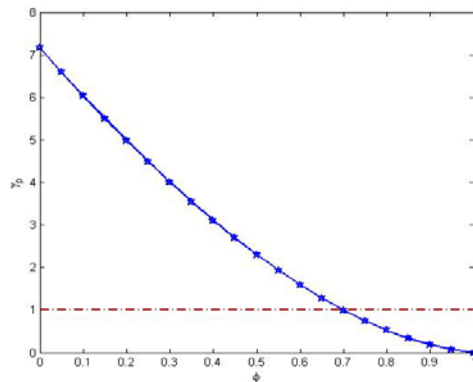


Figure 2 γ_p

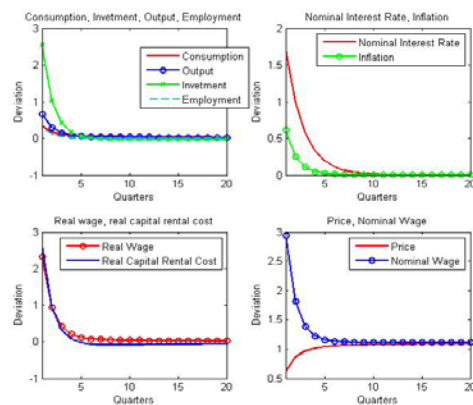


Figure 3 IRF, benchmark setting

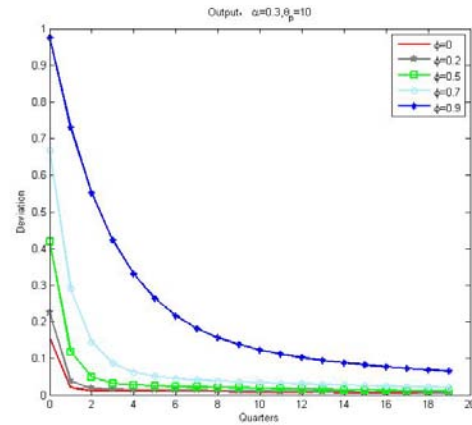


Figure 4 The IRF of output, benchmark setting, ϕ variation

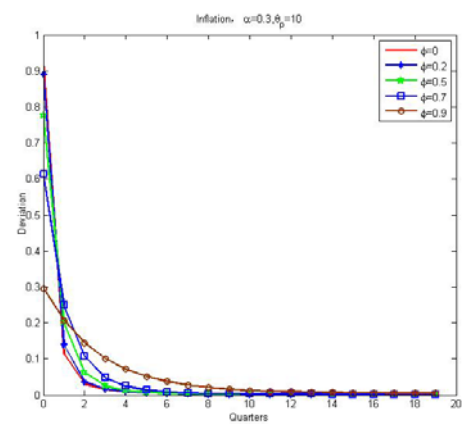


Figure 5 The IRF of inflation, benchmark setting, ϕ variation

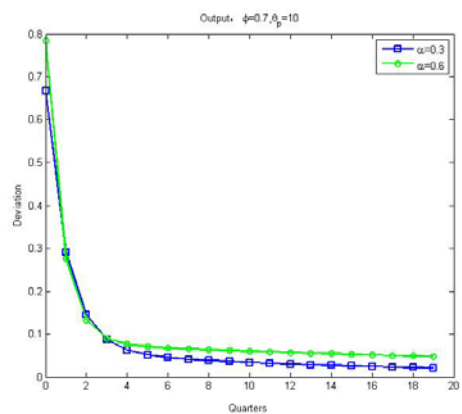


Figure 6 Output, Benchmark Setting, sensitivity analysis of α



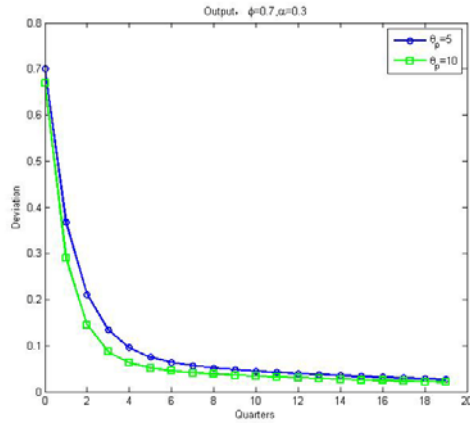


Figure 7 Output, Benchmark Setting, sensitivity analysis of θ_p

Table 7 Contract multiplier, output

ϕ	0	0.2	0.5	0.7	0.9
	0.0735	0.0783	0.1171	0.2168	0.5653

Table 8 Contract multiplier, inflation rate

ϕ	0	0.2	0.5	0.7	0.9
	0.0330	0.0413	0.0803	0.1728	0.4895

7 Conclusion

We estimate the ratio of intermediate input to total output as about 60% , and calibrate intermediate input's output elasticity ϕ in the model as 0.7. Meanwhile , we introduce intermediate input in standard Keynesian sticky price model. This model setting introduces horizontal input-output structure. Both theoretical research and numerical simulation indicate that horizontal input-output structure has important impact on the dynamic reaction of output and inflation under money supply shock. The larger ϕ is, the more persistent output is and the more inneria inflation is. In our benchmark setting, money supply shock has significant and persistent

influence on output and inflation.

In input-output structure, vertical input-output structure is another important feature. Several questions arise: what about China's vertical input-output structure, how to model this structure, and how does it get its mechanism in new Keynesian model, which are potential research in the future.

The value of ϕ does not only indicate the sophistication of the economic production, but also indicate the technology efficiency. Our model depicts the former but ignore the latter. So in order to describe the economy better, considering both the sophistication of production and technology efficiency of technology is necessary, which is also a potential improvement in the future.

Acknowledgment

This work is supported by Shanghai University of Finance and Economics Phd Research Innovation Fund (NO. CXJJ-2008-324)

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