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$\overline{X}$ Charts with Asymmetric Sampling Intervals for Defecting Asymmetric Losses

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Abstract: In many production process, upward and downward shifts of process mean imply different losses. But the most literature don't involved this issue. To meet the need of quality control in these situations, this paper proposed two $\overline{X}$ charts with asymmetric sampling intervals. Numerical results indicate that, comparing with standard Shewhart charts, using asymmetric sampling intervals is more efficient in detecting the shifts in one direction, without ignoring the shifts in the other direction. Using the warning limit may speed up in detecting the shifts in one direction.

Keywords: control chart; asymmetric sampling interval**;** asymmetric loss

1 Introduction

In the production process, for some quality characteristics, the value of observation is greater or less than the target value, which implies different losses for the producers. For example, a product line aims at producing the bearings whose diameter are 5cm. It is clear that the diameter is 5.2cm or 4.8cm for the products not making the same loss. However, up to date, this issue has not been fully reflected in the control charts, and this asymmetry may be very important in terms of the producers in practice. Under this circumstances, it is very necessary to design control charts which have different sensitivity for positive and negative shifts. Without loss of generality, assume that positive shift of the process mean is even more important.

The main idea of an asymmetric sampling intervals chart(**ASI**) is: if the current sample point falls in the upper control zone, the natural inclination in this situation would be to take another sample quickly, such as in 10 minutes, rather than wait the usual an hour for the next sample; but if the sample point falls within the lower zone, it might be reasonable to wait another time intervals for the next sample. Thus the length of the sampling intervals depend on where the current sample is.

The idea of using a variable sampling interval(**VSI**) was firstly proposed by Reynolds et al (1988) for the $\overline{X}$ control chart. Next, they used the **VSI** in CUSUM charts(Reynolds et al,1990). Saccucci et al(1992) proposed the VSI EWMA charts. Baxley(1995)studied the practical application of it.Coast(1997), Prabhu et al(1994) used the variable sampling interval and sample size(**VSS**) in the $\overline{X}$ control chart. G.Celano(1999) integrated the VSI, VSS and run test into the standard Shewhart control chart.Costa(2006), Lin(2007) combined the ideas of varying sample size,varying sampling intervals and varying control limits to the normal process and the non-normal process,respectively. Jensen et al (2008) shows that the impact of parameter estimation on adaptive control charts performance must be taken into consideration.Shi et al(2009) studied a new variable sampling control scheme at fixed times for monitoring the process dispersion. Li(2009) studied adaptive charting schemes based on double sequential probility ratio tests. The results all showed that the use of a **VSI** idea could speed up in the detecting the shifts of the process mean.

The above literatures assumed that it was symmetric about the centrer line in the use of varing sample interval.As a result, it may be inadequate to reflect the working conditions. So this paper used the **VSI** idea that trying asymmetric sampling intervals to cope with the asymmetriy of the loss.

The idea that to give the control charts warning limits can speed up in the detecting the shifts in the process mean, which was used in this paper.

This paper firstly proposed $\overline{X}$ charts with asymmetric sampling intervals and numerical results were given. Then the control charts were added a warning limit in the upper control zone in order to speed up in detecting the upward shifts of the process mean. Finally, we used three kinds of sampling intervals. The initial performance and the steady-state performance all indicated that using asymmetric sampling intervals is more efficient in detecting the shifts in one direction, without missing the shifts in the other direction.

2 The ASI $\overline{X}$ Charts

2.1 The ASI $\overline{X}$ Charts Design

Consider the situation in which the distribution of observations from the process is normal with mean μ and known variance σ^2, where μ_0 denotes the value for the in-control process mean. When some assignable cause occurs, the process mean shifts from μ_0 to $\mu_1 = \mu_0 + \sigma/\sqrt{n}$, where n is the sample size.

Now the standard $\overline{x}$ chart is used for monitoring a process .Assume that random samples of size n are taken,then at each sampling point the mean is computed, and plotted on a control chart with centerline μ_0 and control limits $\mu_0 \pm \gamma\sigma/\sqrt{n}$,where γ is frequently taken to be 3. When the sample mean falls outside the control limits, a signal is given,then the search for the assignable cause is undertaken. Either prove it a false-alarm or remove the assignable cause. The control zone is divided into two parts as follows:

Upper control zone:

$$I_U = [\mu_0, \mu_0 + \gamma\sigma/\sqrt{n}),$$

Lower control zone:

$$I_L = (\mu_0 - \gamma\sigma/\sqrt{n}, \mu_0).$$

The length of the sampling intervals depend on where the current sample is. If the current sample mean falls in the upper control zone I_U , then it is reasonable to take the next sample by waiting the time h_1 in order to improve the capacity in detecting the upward shifts. On the other hand, if the current sample mean falls in the lower control zone I_L , then it is reasonable to take the next sample by waiting the time h_2 . The quality characteristic of the product falling in the upper zone, which gives rise to a larger quality loss. So h_1 will be smaller in order to quickly find the shift which causes a large quality loss. Of cause the first sampling interval should select h_1 to make these charts more sensitive to start-up problems.

2.2 The Statistics Properties of ASI $\overline{X}$ Charts

The statistical properties of variable sampling interval control charts are decided by the time which is needed to give a signal.Average time to signal (ATS) is the exceped value of the time from the start of the process to the time when the chart signals. Adjusted average time to signal ($AATS$)is the expected value of the time from the occurrence of an assignable cause to the time the chart signals.

As long as the control limit is fixed, there is no impact on the probability of $\bar{x}$ falling outside the control limits to change the sampling intervals. Whatever value the sampling interval is, the probability $\bar{x}$ falls outside the control limits is

$$q = \Pr(\overline{X} \le \mu_0 - \gamma\sigma/\sqrt{n} \text{ or } \overline{X} \ge \mu_0 + \gamma\sigma/\sqrt{n}) \quad (1)$$

If N =number of samples after the process shift until a signal,when there is no changes in the process.The distribution of N is geometric with parameter q .So the average run length（ ARL ）is

$$E(N) = 1/q \quad (2)$$

Let R_i be the sampling interval used before the i th sample,then

$$R_i = h(\bar{x}_{i-1}) = \begin{cases} h_1, & if \quad \bar{x}_{i-1} \in I_U \\ h_2, & if \quad \bar{x}_{i-1} \in I_L \end{cases} \quad (3)$$

2.2.1 Initial Performance Measure of Charts

Suppose that the process mean changed at 0 time.

If T =the time untile giving a signal，then $T = \sum_{i=1}^{N} R_i$.

As the process mean is fixed ,it follows that R_1 , $R_2, \cdots R_N$ is independent and identical distribution.According to Wald Equation,we obtain

$$ATS = E(N)E(R_i) \quad (4)$$

Let $p_U = p(h(\bar{x}) = h_1) = p(\bar{x} \in I_U)$, $p_L = p(h(\bar{x}) = h_2) = p(\bar{x} \in I_L)$, $j = 1,2$, then

$$E(R_i) = h_1 \frac{p_U}{1-q} + h_2 \frac{p_L}{1-q} \quad (5)$$

where the probability is a conditional probability when the chart doesn’t alarm, and $p_U + p_L = 1 - q < 1$. Then we obtain

$$ATS = \frac{p_U h_1}{q(1-q)} + \frac{p_L h_2}{q(1-q)} \quad (6)$$

where q and p_U , p_L depend on the value of μ ,and the probabilities under μ_0 will be denoted by q_0 and p_{0U} , p_{0L} and the probabilities under μ_1 by q_1 and p_{1U} , p_{1L} .

2.2.2 Steady-state Performance Measure of Control Charts

In practice it is more usual that the shift in the process mean doesn’t occur at the beginning but at some random time in the future. For such case the appropriate measure to the statistics performance of a control chart is $AATS$,that is, the average length of time from shift to signal.

Suppose 1: the probability of which the process mean change happens at one interval is in proportion of the product of which the length of this interval and the probability what the process in control of the interval .

Suppose 2: the time between the process mean getting changed and the next sample follows uniform distribution. Then, we have

$$AATS = E(Y) + [E(N) - 1]E(R_i)$$
$$= \frac{h_1^2 p_{0U} + h_2^2 p_{0L}}{2(h_1 p_{0U} + h_2 p_{0L})} + \frac{h_1 p_{1U} + h_2 p_{1L}}{q_1} \quad (7)$$

where Y is the time from the process shift until the next sample.

2.3 The Results of ASI $\overline{X}$ Charts

In order to control the cost of sampling and inspection, the general assumption is that fixed sampling resources are used when the process is in-control state, that is, fixed-sample-size samples are taken under the conditions of using fixed sampling intervals. Design **ASI** $\overline{X}$ charts to make $E(R)$ equal to a fixed sampling interval h_0 when $\mu = \mu_0$.

When $\mu = \mu_0$,there is

$$E(R \mid \mu = \mu_0) = \frac{h_1 p_{0U} + h_2 p_{0L}}{1 - q_0} \quad (8)$$

where

$$p_{0U} = \mathrm{p}(\mu_0 < \overline{X} < \mu_0 + \gamma\sigma / \sqrt{n} \mid \mu = \mu_0) ,$$
$$p_{0L} = \mathrm{p}(\mu_0 - \gamma\sigma / \sqrt{n} < \overline{X} < \mu_0 \mid \mu = \mu_0) .$$

So we obtain $p_{0U} = p_{0L} = (1 - q_0)/2$ and $h_1 + h_2 = 2h_0$.

The value of h_0 has no effect on the analysis of results,for ease of comparison,we select $h_0 = 1$.

Table 1 gives different combinations of sampling intervals (h_1, h_2) and the values of ATS and $AATS$ under a range of δ .

From table 1 ,we can see ,

(1) Just as intuition, the smaller value of h_1 , the faster upward shifts of the process are detected. With the increase in the values of h_1 and the decrease in the values of h_2 , the speed differences become smaller in detecting shifts in different directions. When $h_1 = h_2 = h_0$, the asymmetric sampling intervals control charts are degraded into standard Shewhart charts. If absolute value of the shifts magnitude in the process mean is the same value, the detecting speed is also the same.

(2) The speed of **ASI** charts is significantly faster than that of the Shewhart charts in detecting the upward shifts of the process mean,and the larger shifts magnitude ,the more obvious effect.

(3) The faster speed in detecting the positive shifts of the process mean is at the cost of the lower speed in detecting the negative shifts.

3 $\overline{X}$ Charts with A Warning Limit （WASI）

3.1 The Description of WASI $\overline{X}$ Charts

In order to speed up in detecting the upward shifts of the process mean, the control charts are added a waring limit $\mu_0 + k\sigma/\sqrt{n}$ between the center limit μ_0 and the upper control limit $\mu_0 + \gamma\sigma/\sqrt{n}$, where $0 \le k < \gamma$. So the whole control zone are divided into three parts:

the upper warning zone

$$I_1 = [\mu_0 + k\sigma/\sqrt{n}\ ,\ \mu_0 + \gamma\sigma/\sqrt{n})$$

the upper center zone

$$I_2 = [\mu_0\ ,\ \mu_0 + k\sigma/\sqrt{n})$$

the lower control zone

$$I_3 = (\mu_0 - \gamma\sigma/\sqrt{n}\ ,\ \mu_0)$$

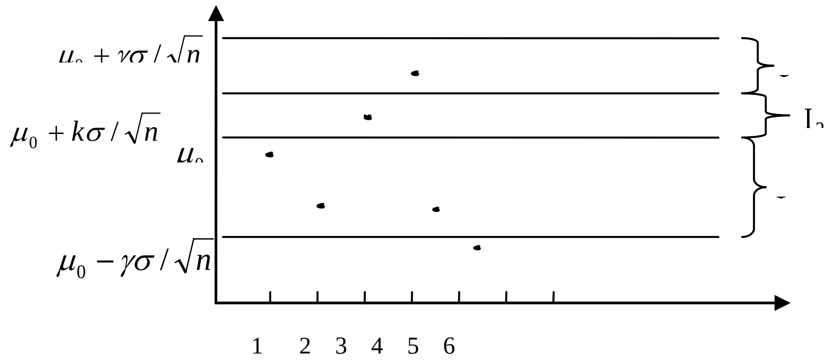

Fig1 the asymmetric sampling intervals control chart with a warning limit

Table 1 The comparisons of asymmetric sampling interval $\overline{X}$ charts and the matched FSI $\overline{X}$ charts

$\frac{\sqrt{n}(\mu-\mu_0)}{\sigma}$	The initional performance				The steady-state performance			
	(0.1,1.9)	(0.5,1.5)	(0.8,1.2)	(1.0,1.0)	(0.1,1.9)	(0.5,1.5)	(0.8,1.2)	(1.0,1.0)
0.0	370.40	370.40	370.40	370.40	370.30	370.20	369.92	369.90
0.5	102.22	125.78	143.45	155.22	102.47	125.59	143.04	154.72
1.0	17.21	29.07	37.97	43.89	17.73	29.04	37.62	43.39
1.5	3.42	8.56	12.40	14.97	4.10	8.61	12.09	14.47
2.0	0.94	3.32	5.11	6.30	1.69	3.42	4.82	5.80
3.0	0.21	1.01	1.60	2.00	1.01	1.13	1.32	1.50
4.0	0.12	0.59	0.95	1.19	0.92	0.72	0.67	0.69
-0.5	208.23	184.67	167.00	155.22	207.79	184.10	166.45	154.72
-1.0	70.58	58.72	49.82	43.89	69.87	58.00	49.21	43.39
-1.5	26.51	21.38	17.53	14.97	25.64	20.58	16.88	14.47
-2.0	11.67	9.28	7.50	6.30	10.72	8.44	6.83	5.80
-3.0	3.79	2.99	2.40	2.00	2.80	2.12	1.72	1.50
-4.0	2.26	1.78	1.43	1.19	1.26	0.91	0.75	0.69

Note：The above table uses $\gamma = 3$， and $(\cdot,\cdot)$ is the value of (h_1, h_2) .

Sampling rules: each time collect a sample with a fixed size n, and then calculate the sample mean $\overline{X}$, if $\overline{X}$ falls within the zone I_1, a short sample interval d_1 is chosed as the next sampling interval to improve the charts in detecting the upward shifts ; if $\overline{X}$ falls within the zone I_2, we choose a longer sampling interval d_2; if $\overline{X}$ falls within the lower control zone I_3, we choose a medium length d_3 as the sampling interval;if $\overline{X}$ falls outside the control zone, a signal is given.The sampling function is marked for $d(\overline{X})$.

Let R_i be the sampling interval used before the i th sample,then

$$R_i = d(\bar{x}_{i-1}) = \begin{cases} d_1, & if \quad \bar{x}_{i-1} \in I_1, \\ d_2, & if \quad \bar{x}_{i-1} \in I_2, \\ d_3, & if \quad \bar{x}_{i-1} \in I_3. \end{cases} \tag{9}$$

3.2 Statistics Efficiency Measure of WASI Charts

3.2.1 Initial Performance Measure of Charts

Suppose that the process mean changed at 0 time. As the process mean is fixed ,it follows that R_1 , $R_2, \cdots$ R_N is independent and identical distribution.According to Wald Equation,we obtain

$$ATS = E(N)E(R_i) \tag{10}$$

Let

$$p_j = \Pr(d(\bar{x}) = d_j) = \Pr(\bar{x} \in I_j) \quad j = 1,2,3,$$

then

$$E(R_i) = d_1 \frac{p_1}{1-q} + d_2 \frac{p_2}{1-q} + d_3 \frac{p_3}{1-q} \tag{11}$$

where the probability is a conditional probability, at the time the chart doesn't alarm, and we have $p_1 + p_2 + p_3 = 1 - q < 1$. Then we can obtain

$$ATS = \frac{p_1 d_1}{q(1-q)} + \frac{p_2 d_2}{q(1-q)} + \frac{p_3 d_3}{q(1-q)} \tag{12}$$

where q and p_j depend on the value of μ ,and the probabilities under μ_0 will be denoted by q_0 and p_{0j} and the probabilities under μ_1 by q_1 and p_{1j}, $j = 1,2,3$.

3.2.2 Steady-state Performance Measure of Control Charts

In practice it is more usual that the shift in the process mean doesn't occur at the beginning but at some random time in the future.For such case the appropriate measure to the statistics performance of a control chart is **AATS**,that is,the average length of time from shift to signal.

The same assumption as（2.2.2），we obtain

$$AATS = E(Y) + [E(N) - 1]E(R_i)$$
$$= \frac{d_1^2 p_{01} + d_2^2 p_{02} + d_3^2 p_{03}}{2(d_1 p_{01} + d_2 p_{02} + d_3 p_{03})}$$
$$+ \frac{d_1 p_{11} + d_2 p_{12} + d_3 p_{13}}{q_1} \tag{13}$$

where Y is the time from the process shift until the next sample.

3.3 The Comparisons of WASI and Shewhart Charts and Sensitivity Analysis

In evaluating the usefulness of the **WASI** $\bar{X}$ charts, it seems natural to compare the the performance of the **WASI** $\bar{X}$ charts to the standard Shewhart. i.e.**FSI** $\bar{X}$ charts. If the two charts had the same control limits $\mu_0 \pm \gamma\sigma / \sqrt{n}$, then they should have the same value of q and average run length, which means that the sampling intervals **WASI** $\bar{X}$ charts use don't affect the samples numble until a signal is given.

Conveniently, let the length of sampling interval used by a fixed sampling interval (FSI)lchart be one unit of time. For example, if one sample is checked every 2 hours in a fixed sampling interval chart, then let two hours be one time unit. So the value of ATS of a FSI $\bar{X}$ chart is equal to the value of its average run length. Design a **WASI** $\bar{X}$ chart to make the length of an average sampling interval $E(R)$ in control state($\mu = \mu_0$) equal to one time unit , then during an in-control period the two controls have the same value of **ATS,**that is,the two charts are matched with each other.

Compare the values of **ATS** of two different charts based on μ_1 （ $\mu_1 \neq \mu_0$ ） in order to determine which chart is better in detecting the shifts of the process mean.

Using three sampling intervals, when $\mu = \mu_0$,requires $E(R_i) = 1$,which means

$$d_1 p_{01} + d_2 p_{02} + d_3 p_{03} = 1 - q_0 \qquad (14)$$

For the fixed value of γ， given the values of d_1, d_2 and d_3， we obtain from (14):

$$k = \Phi^{-1}\left(\frac{1 - q_0 + 0.5d_2 + 0.5d_3 - d_1\Phi(\gamma) - d_3\Phi(\gamma)}{d_2 - d_1}\right)$$

and the three zones I_1 , I_2 and I_3 also are obtained.From

$d_1 p_{01} + d_2 p_{02} + d_3 p_{03} = 1 - q_0$,

$p_{01} + p_{02} + p_{03} = 1 - q_0$,

$2p_{03} = 1 - q_0$,

and $p_{01} > 0, \quad p_{02} > 0$， we obtain

$0 < d_1 < 2 - d_3 < d_2$.

When $\mu = \mu_0 + \delta\sigma/\sqrt{n}$， there is

$$q_1 = 2 - \Phi(\gamma - \delta) - \Phi(\gamma + \delta),$$

$$p_{11} = \Phi(\gamma - \delta) - \Phi(k - \delta),$$

$$p_{12} = \Phi(k - \delta) + \Phi(-\delta)$$

$$p_{13} = \Phi(-\delta) - \Phi(-\gamma - \delta) \quad .$$

Table2，3 give the initial performance and the steady-state performance of WASI $\overline{X}$ charts, respectively, based on the values of δ under the different combinations of sampling intervals.

The results given in tables 2,3 show the following:

1． $d_3 = 1$ means that **WASI** charts use the same sampling intervals as FSI charts, when the sample mean falls between the center line and the lower control limit. The **WASI** chart is consistently faster than the Shewhart chart for the upward shifts, especially, for the greater upward shifts, the value of its **ATS** is much smaller than that of the Shewhart chart. The efficiency is improved very obviously. However, for the small or medium downward shifts of the process mean, the performance of the Shewhart chart is better; when great downward shifts ($\delta < -2.5$) occur, the speed in detecting downward shifts of **WASI** charts are no significent diffierents, because of the probability of a sample mean falling under the center line is $p(\mu_0 \leq \overline{X}) = \Phi(\delta)$ at this time. For example, when $\delta = 3$, $p(\mu_0 \leq \overline{X})$ $= 0.99865$， the samples almost fall under the center line, so almost $d_3 = 1$ is at work, just as the standard Shewhart $\overline{X}$ chart is at work. This makes upward shifts detected rapidly without missing downward shifts.

2． The bigger value of d_3 , the faster speed in detecting the upward shifts of the process mean , at the same time, the lower speed in detecting the downward shifts we get.

3. The value of d_1 has a great effect on the statistic efficiency of WASI charts. The smaller value of d_1 , the greater effect on the statistical efficiency of WASI charts. Because the small value of d_1 can quickly respond to the upward shifts of the process mean, and this can speed up in detecting the upward shifts.

Table 2 The comparisons of WASI $\overline{X}$ charts and matched FSI $\overline{X}$ charts（the initial performance）

$\frac{\sqrt{n}(\mu-\mu_0)}{\sigma}$	FSI $\bar{x}$	$d_3=1.0$			$d_3=1.2$			$d_3=1.5$		
		(0.1,1.9)	(0.5,1.9)	(0.5,1.5)	(0.1,1.9)	(0.5,1.9)	(0.5,1.5)	(0.1,1.9)	(0.5,1.9)	(0.5,1.5)
0.0	370.39	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40	370.40
0.5	155.22	131.96	136.15	142.30	123.13	125.35	134.04	112.47	125.78	125.78
1.0	43.89	28.02	30.20	35.07	24.54	25.62	31.89	20.63	29.07	29.07
1.5	14.97	6.42	7.25	10.22	5.38	5.75	9.28	4.29	8.56	8.56
2.0	6.30	1.72	2.01	3.75	1.42	1.55	3.50	1.14	3.32	3.32
3.0	2.00	0.27	0.30	1.04	0.24	0.25	1.02	0.22	1.01	1.01
4.0	1.19	0.12	0.13	0.59	0.12	0.12	0.60	0.12	0.59	0.59
-0.5	155.22	164.71	162.26	160.51	175.91	174.41	171.34	191.07	184.67	184.67
-1.0	43.89	46.49	45.72	45.33	52.34	51.84	51.08	60.61	58.72	58.72
-1.5	14.97	15.50	15.33	15.26	18.07	17.95	17.81	21.80	21.38	21.38
-2.0	6.30	6.40	6.37	6.36	7.60	7.58	7.55	9.37	9.28	9.28
-3.0	2.00	2.00	2.00	2.00	2.40	2.40	2.40	3.00	2.99	2.99
-4.0	1.19	1.19	1.19	1.19	1.43	1.43	1.43	1.78	1.78	1.78
k	0.67	0.92	0.67	0.51	0.67	0.38	0.28	0.00	0.00	0.00

Table 3 The comparisons of WASI $\overline{X}$ charts and matched FSI $\overline{X}$ charts（the steady-state performance）

δ	FSI $\bar{x}$	$d_3=1.0$			$d_3=1.2$			$d_3=1.5$		
		(0.1,1.9)	(0.5,1.9)	(0.5,1.5)	(0.1,1.9)	(0.5,1.9)	(0.5,1.5)	(0.1,1.9)	(0.5,1.9)	(0.5,1.5)
0.5	154.72	131.81	140.00	141.94	123.05	133.15	133.75	112.51	125.59	125.59
1.0	83.39	28.08	33.95	34.84	24.70	31.49	31.74	20.93	29.04	29.04
1.5	14.47	6.69	9.83	10.10	5.73	9.18	9.24	4.77	8.61	8.61
2.0	5.80	2.15	3.68	3.72	1.91	3.52	3.51	1.72	3.42	3.42
3.0	1.50	0.84	1.13	1.08	0.83	1.11	1.08	0.88	1.13	1.13
4.0	0.69	0.72	0.71	0.66	0.73	0.70	0.67	0.78	0.72	0.72
-0.5	154.72	164.39	161.46	160.04	175.49	171.34	170.81	190.60	184.10	184.10
-1	83.39	46.13	45.36	44.87	51.86	50.69	50.49	59.99	58.00	58.00
-1.5	14.47	15.17	14.96	14.81	17.57	17.26	17.19	21.11	20.58	20.58
-2.0	5.80	6.09	5.98	5.91	7.11	6.96	6.93	8.65	8.44	8.44
-3.0	1.50	1.70	1.61	1.56	1.91	1.80	1.77	2.26	2.12	2.12
-4.0	0.69	0.89	0.80	0.75	0.94	0.83	0.80	1.05	0.91	0.91
k		0.67	0.46	0.67	0.51	0.27	0.38	0.28	0.00	0.00

Note：The above two tables use $\gamma=3$， and $(\cdot,\cdot)$ is the value of (d_1,d_2) .

4．The better initial performance of control charts in detecting the upward shifts is, the greater difference between d_2 and d_1 with d_3 fixed. While the smaller value of d_1, the better the steady-state performance.

5．Table2，3 chose the forms matched with the table 1, that is, if the sample falls within the lower control zone,we chose 1.0, 1.2 and 1.5 as the length of sampling intervals respectively. Results show that adding a warning limit may speed up in detecting the upward shifts of the process mean , and the effect is very evident. For example,when $(d_1,d_2,d_3)=(0.1,1.9,1.5)$ and $\delta=1.5$, there is $ATS=4.29$, but

$ATS = 8.56$ corresponding to no warning limit .

4 Conclusions

Based on variable sampling intervals,this paper designes asymmetric sampling intervals $\overline{X}$ charts. The results showed that, comparing with standard Shewhart charts, using asymmetric sampling intervals is more efficient in detecting the shifts in one direction, without ignoring the shifts in the other direction,and using the warning limit may speed up in detecting the shifts in one direction.A similar chart can be designed when it is interested of the downward shifts.

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References

[1] Baxley.R.V..JR.. An application of variable sampling interval control charts [J].Journal of Quality Technology,1995, 27:275-282.

[2] Celano.G,·Costa, A.F.B. Statistical design of variablesample size and sampling interval $\overline{X}$ control charts with run rules [J]. Int J Adv Manuf Technol，2006，28:966-977.

[3] Costa,A.F.B, $\overline{X}$ charts with variable parameters[J]. Journal of Quality Technology, 1999, 31:408-416.

[4] Costa,A.F.B. $\overline{X}$ charts with variable sample size and sampling intervals[J]. Journal of Quality Technology, 1997,29:197-204.

[5] Jensen W A,Bryce G R,and Reynolds.Jr MR. Design issues for adaptive control charts[J].Quality and Reliability Engineering International,2008,24: 429-445.

[6] Lihui Shi,Changliang Zou,Zhaojun Wang and Kapur K.C. A new variable sampling control scheme at fixed times for monitoring the process dispersion[J]. Quality and Reliability Engineering International, 2009,25: 961-972.

[7] Park.C and Reynolds,JR. Economic design of a variable sampling rate $\overline{X}$ chart[J].Journal of Quality Technology,1999,31(4):427-443.

[8] Prabhu,S.S.,Montgomery,D.C,Runger,G.C.A combined adaptive sample size and sampling intervals $\overline{X}$ control scheme[J]. Journal of Quality Technology, 1994, 26:164-176.

[9] Reynolds.Jr., M.R., Amin, R.W., Arnold,J.C. Cusum charts with variable sampling intervals[J]. Technometrics, 1990, 32: 371-384.

[10] Reynolds.Jr., M.R., Amin, R.W., Arnold,J.C.,and Nachlas, J.A.. $\overline{x}$ charts with variable sampling intervals[J].Technometrics, 1988, 30: 181-192.

[11] Saccucci,M.S, Amin, R.W., Lucas,J.M. Exponentially weighted moving average control schemes with variable sampling intervals[J]. Communications in Statistics-Simulation and Computation, 1992, 21:627-657.

[12] Yan Li,Xiaolong Pu and Fugee Tsung. adaptive charting schemes based on double sequential probility ratio tests[J].2009,21-39.

[13] Yu-Change Lin, Chao-Yu Chou. Non-normality and the variable parameters x-bar charts [J]. European Journal of Operational Research, 2007,176:361-373.

[14] Yu-Change Lin, Chao-Yu Chou. the variable sampling rate x-bar control charts for monitoring autococcelated process[J]. .Quality and Reliability Engineering International,2008, 24:855-870.