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Estimation of Efficiency and Varying-Elasticity with DEA Model

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***Abstract*:** The conventional economic growth model requires unrealistically strong assumptions, such as competitiveness of factor markets, Cobb-Douglas form of the underlying aggregate production function, and the efficiency of the technique. Without these restrictive assumptions, DEA involves the use of linear programming methods to construct a non-parametric piecewise frontier over the data. This paper puts forward that the nonparametric DEA model can not only be applied to calculate efficiencies but also to estimate varying-elasticity of output relative to the surface.

***Keywords*:** Efficiency; Varying-elasticity; Production function; Data envelope analysis

I. INTRODUCTION

Production functions are a fundamental component of all economics. Production functions relate productive inputs (e.g. capital, labor) to outputs, reflect the effect and influence of production factor on output at certain technological conditions. Cobb-Douglas production function is preferred for its simple structure, meaningful parameter and easy estimation. Assuming two factors production, physical capital (K) and labor (L), the Cobb-Douglas production function is as follow:

$$Y = AK^{\alpha}L^{\beta} \qquad (1)$$

The parameters α and β, stand for the elasticity of capital and labor to output, measure how the amount of output Y responds to changes in the input.

However, the conventional economic growth model such as Cobb-Douglas production function requires unrealistically strong assumptions, such as competitiveness of factor markets, Cobb-Douglas form of the underlying aggregate production function, and the efficiency of the technique.

Modern efficiency measurement begins with Farrell (1957) who drew upon the work of Debreu (1951) and Koopmans (1951) to define a simple measure of firm efficiency which could account for multiple inputs. Frontiers have been estimated using many different methods over the past 50 years. The two principal methods are: data envelopment analysis (DEA) and stochastic frontiers, which involve mathematical programming and econometric methods, respectively.

DEA involves the use of linear programming methods to construct a non-parametric piecewise frontier over the data, so as to be able to calculate efficiencies relative to this surface. However, it's widely accepted that the DEA model is not been able to estimate elasticity of output due to the lack of function form. This paper puts forward that the nonparametric DEA model can not only be applied to calculate efficiencies but also to estimate varying-elasticity of output relative to the surface.

The paper is organized as follows. In the following section we will describe specification and estimation method in DEA model. Section III contains a description of our data, and our empirical results are given and discussed in this section. The final section contains concluding remarks.

II. ESTIMATION OF EFFICIENCY AND ELASTICITY

Efficiency measurement has been a subject of tremendous interest as organizations have struggled to improve productivity. Reasons for this focus were best stated fifty years ago by Farrell (1957) in his classic paper on the measurement of productive efficiency. Twenty years after Farrell's seminal work, and building on those ideas, Charnes et al. (1978), responding to the need for satisfactory procedures to assess the relative efficiencies of multi-input multi-output production units, introduced a powerful methodology which has subsequently been titled data envelopment analysis (DEA). Since the advent of DEA in 1978, there has been an impressive growth both in theoretical developments and applications of the ideas to practical situations. Banker et al. (1984) (BCC), extended the earlier work of Charnes et al. (1978) by providing for variable returns to scale (VRS). Wade D et al (2009) provide a sketch of some of the major research thrusts in data envelopment analysis (DEA) over the past three decades.

The CRS assumption is only appropriate when all DMU's are operating at an optimal scale (i.e one corresponding to the flat portion of the LRAC curve). Imperfect competition, constraints on finance, etc. may cause a DMU to be not operating at optimal scale. Banker, Charnes and Cooper(1984) suggested an extension of the CRS DEA model to account for variable returns to scale (VRS) situations. The use of the CRS specification when not all DMU's are operating at the optimal scale will result in measures of TE which are confounded by *scale efficiencies* (SE). The use of the VRS specification will permit the calculation of TE devoid of these SE effects.

The VRS linear programming problem can be provide as:

$$
\begin{aligned}
&\text{Max}_{\theta,\lambda}\ \theta,\\
&\text{st}\quad -y_i + Y\lambda \geq 0,\\
&\theta x_i - X\lambda \geq 0,\\
&N1'\lambda = 1\\
&\quad \lambda \geq 0, \qquad (2)
\end{aligned}
$$

where N1 is an N×1 vector of ones. This approach forms a convex hull of intersecting planes which envelope the data points more tightly than the CRS conical hull and thus provides technical efficiency scores which are greater than or equal to those obtained using the CRS model. The VRS specification has been the most commonly used specification in the 1990's.

Many studies have decomposed the TE scores obtained from a CRS DEA into two components, one due to scale inefficiency and one due to "pure" technical inefficiency. This may be done by conducting both a CRS and a VRS DEA upon the same data. If there is a difference in the two

TE scores for a particular DMU, then this indicates that the DMU has scale inefficiency, and that the scale inefficiency can be calculated from the difference between the VRS TE score and the CRS TE score. Figure1 illustrate this theory.

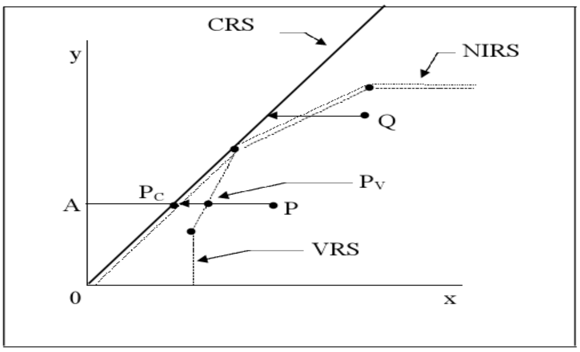

Figure 1 Calculation of Scale Economies in DEA

The elasticity of capital and labor to output, measure how the amount of output Y responds to changes in the input. Iwata, Khan, and Murao (2003) , Bing Xu, Berlin Wu (2007) applied a nonparametric method to estimate the elasticity of capital and labor. However, it's widely accepted that the DEA model is not been able to estimate elasticity of output due to the lack of function form. In this paper, we will prove that the nonparametric DEA model can not only be applied to calculate efficiencies but also to estimate varying-elasticity of output relative to the surface.

Take

$X^0 = (x^0_1, x^0_2, \cdots, x^0_n);$

$Y^0 = \min\{Y_j, Y_j \neq 0, 1 \le j \le n\}$

$x^1_i = x_i + \Delta x_i$

$X^1 = (x^0_1, x^0_2, \cdots, x^1_i, \cdots, x^0_n)$

Denote θ_0 and θ_1 as the optimal value of the model (2) when (X_{n+1}, Y_{n+1}) = (X^0, Y^0) and (X_{n+1}, Y_{n+1}) = (X^1, Y^0) respectively. Then, $y_1 = \theta_0 Y_0$ and $y_2 = \theta_1 Y_0 r$ can be taken as the optimal value of output at the points X^0 and X^1 respectively.

According to the definition of elasticity, the elasticity of capital can be estimated as:

$$\alpha_K = \frac{\partial Y}{\partial K}\frac{K}{Y} = \frac{\partial Y / Y}{\partial K / Y} \approx \frac{\ln[(Y+\Delta Y)/Y]}{\ln[(K+\Delta K)/K]} = \frac{\ln(\theta_1 Y/\theta_0 Y)}{\ln[(K+\Delta K)/K]} = \frac{\ln(\theta_1/\theta_0)}{\ln(1+\Delta K/K)} \tag{3}$$

III. DATA AND EMPIRICAL RESULTS

The main variables contain Gross Domestic product (Y), Capital (K) and Labour force (L). We choose the 28 provinces of China at the year 2007. In order to eliminate the influence of inflation, we calculate the true data on the base year of 1990. Gross Domestic Product, which stands for output in the paper, is calculated by expenditure approach. The number of labour force is calculated by total employed persons at the year-end. In this paper, we follow Jun Zhang (2002) to measure the capital.

The results of the efficiencies and the varying-elasticity of capitals, estimated by computer program of DEAP version 2.1, are showed in table 1, figure 1 and 2.

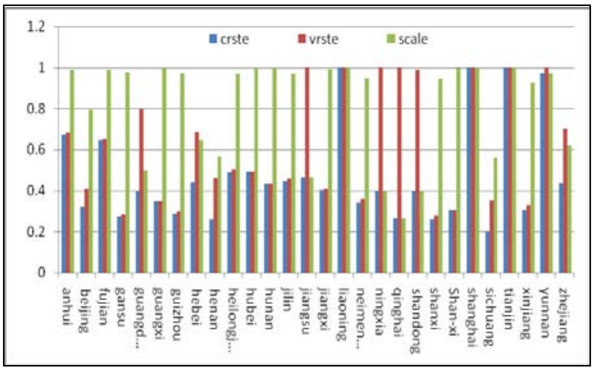

Figure2 the efficiency results

Table 1 efficiency summary

Provinces	*crste*	*vrste*	*scale*	
anhui	0.673	0.681	0.989	irs
beijing	0.323	0.408	0.793	drs
fujian	0.645	0.652	0.989	irs
gansu	0.274	0.282	0.974	irs
guangdong	0.398	0.799	0.498	drs
guangxi	0.348	0.349	0.997	irs
guizhou	0.288	0.297	0.97	irs
hebei	0.443	0.684	0.648	drs
henan	0.26	0.461	0.565	drs
heilongjiang	0.486	0.503	0.967	irs
hubei	0.491	0.491	1	-
hunan	0.434	0.434	0.999	-
jilin	0.444	0.458	0.968	irs
jiangsu	0.466	1	0.466	drs
jiangxi	0.403	0.407	0.992	irs
liaoning	1	1	1	-
neimenggu	0.341	0.36	0.948	drs
ningxia	0.399	1	0.399	irs
qinghai	0.263	1	0.263	irs
shandong	0.394	0.986	0.4	drs
shanxi	0.261	0.277	0.943	drs
Shan-xi	0.306	0.307	0.995	irs
shanghai	1	1	1	-
sichuang	0.199	0.354	0.563	drs
tianjin	1	1	1	-
xinjiang	0.305	0.329	0.927	irs
yunnan	0.97	1	0.97	irs
zhejiang	0.436	0.702	0.621	drs

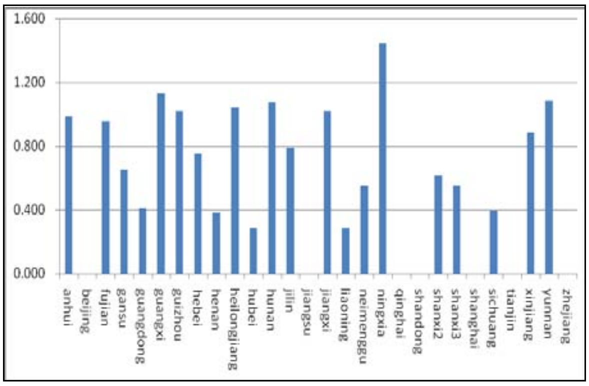

Figure 3 the varying-elasticityof capital

We come to the following conclusions with the Table 1 and figure 2: (1) six provinces, Jiangsu, ningxia, qingha, shanghai, Tianjin, and Yunnan, are under the adequate efficiency. (2) thirteen provinces, Anhui, Fujian, gansu, guangxi, guizhou, Heilongjiang, jilin, Jiangxi, ningxia, Qinghai, Shan-xi, xinjiang, Yunnan, are under the increasing return of scale. Ten provinces, Beijing, Guangdong, hebei, henan, Jiangsu, neimenggu, Shandong, shanxi, sichuang, Zhejiang, are under the decreasing return of scale. Five provinces, Hubei, hunan, Liaoning, shanghai, and Tianjin, are under the optimal scale of return.

Our DEA model allows the elasticity of capital to vary over the entire observed range. Figure 3 shows the elasticity of capital for 28 provinces: Ten provinces are Larger than 0.8, that is, Ningxia, guangxi, Yunnan, hunan, Heilongjiang, guizhou, Jiangxi, anhui, Fujian, and xinjiang. Seven provinces are between 0.4 and 0.8, that is, Jilin, hebei, gansu, shanxi, neimenggu, shan-xi, and Guangdong. Eleven provinces are Smaller than 0.4, that is, Sichuang, Henan, hubei, Liaoning, Beijing, Jiangsu, Qinghai, Shandong, shanghai, Tianjin, and Zhejiang.

We have to point out that, the piecewise linear form of the non-parametric frontier in DEA can cause a few difficulties in efficiency measurement. The problem arises because of the sections of the piecewise linear frontier which run parallel to the axes which do not occur in most parametric functions. if one could reduce the amount of input used and still produce the same output. The input slack problem is often aroused. For example, it’s found in this paper that two provinces have capital slacks (Beijing, Shandong), and ten provinces have labor slacks (Anhui, Guangdong, guangxi, hebei, henan, hubei, hunan, Shandong, sichuang, and Zhejiang).

IV. CONCLUTIONS

The conventional economic growth model requires unrealistically strong assumptions, such as competitiveness of factor markets, Cobb-Douglas form of the underlying aggregate production function, and the efficiency of the technique. Without these restrictive assumptions, DEA involves the use of linear programming methods to construct a non-parametric piecewise frontier over the data. This paper puts forward that the nonparametric DEA model can not only be applied to calculate efficiencies but also to estimate varying-elasticity of output relative to the surface. Our DEA model allows the elasticity of capital to vary over the entire observed range.

We find out the below conclusions: Firstly, six provinces are under the adequate efficiency. Secondly, thirteen provinces are under the increasing return of scale. Ten provinces are under the decreasing return of scale. Five provinces are under the optimal scale of return. Thirdly, Ten provinces are Larger than 0.8, Seven provinces are between 0.4 and 0.8, and Eleven provinces are Smaller than 0.4.

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REFERENCES

[1] Bing Xu, Berlin Wu, On nonparametric estimation for the growth of total factor productivity: A study on china and its four eastern provinces, *International Journal of Innovative Computing, Information and Control*, Volume 3, Number 1, February 2007 pp. 141—150

[2] Charnes , A. , Cooper , W. W. , Rhodes , E. Measuring the Efficiency of Decision Making Units[J] . *European Journal of Operational Research*, 1978, 2:429 - 444.

[3] Debreu, G., “The Coefficient of Resource Utilisation”, *Econometrica*, 1951, 19, 273-292.

[4] Fare , R. , Lovell , C. A. K.Measuring the Technical Efficiency of Production[J] . *Journal of Economic Theory*, 1978, 19:1 50 -162.

[5] Farrell, M. J., Measurement of Productive Efficiency [J], *Journal of the Royal Statistical Society*, 1957, Part3, Serial A.

[6] Koopmans, T. C. An Analysis of Production as an Efficient Combination of Activities [J]. In: T. C. Koopmans (Ed.), Activity Analysis of Production and Allocation , Cowles Commission for Research in Economics. Monograph No. 13, Wiley, NewYork, 1951.

[7] Shigeru Iwata,Mohsin S.Khan and Hiroshi murao, Sources of Economic Growth in East Asia: A Nonparametric Assessment [J], *IMF Staff Papers*, 2003,Vol.50

[8] Solow. R.W. Technical Change and the Aggregate Production Function, *Review of Economics of Statistics*, 1957 (3):312-320.

[9] Wade D. Cook, Larry M. Seiford, Data envelopment analysis (DEA) – Thirty years on, *European Journal of Operational Research* 2009(192) 1–17