COMPARISON OF LONG MEMORY AND ASYMMETRY CAPTURING PERFORMANCE FOR GARCH AND FIGARCH MODELS WITH NIG DISTRIBUTION FOR THAI EXCHANGE RATES

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ABSTRACT. *The purpose of this research is to construct GARCH and FIGARCH model for daily exchange rate returns with Normal, Student's-t and Normal Inverse Gaussian (NIG) error distributions. A model is constructed by extending GARCH-NIG model to FIGARCH-NIG model to study the hyperbolic memory and time variation in the conditional volatility of daily exchange rate returns. It is found that, FIGARCH-NIG model is efficient in capturing hyperbolic memory and time variation in the conditional volatility and the result reveals asymmetric distribution in dollar exchange rate and symmetric distribution in yen exchange rate.*

Keywords: FIGARCH; NIG Distribution; Hyperbolic Memory; Conditional Volatility; Exchange Rate

1. Introduction. Generally, econometric model is assumed to be ceteris paribus with the variance and error term as constant terms. Enders (1995) showed that the economic data which is time series data has high variance with non-stationary variance and error term. The time series data contradict the assumption above and the investors simply focus on the conditional variance such as the prediction of return and variance. Therefore, ARCH (Autoregressive Conditional Heteroscedastic) was developed and applied to ARMA (Autoregressive Moving Average) model in order to correct the model and show the value of mean and variance simultaneously (Engle (1982)). ARCH was continuously developed to GARCH (Generalized Autoregressive Conditional Heteroscedastic) to adjust the variance to characterize as ARMA process which is applied in time – variance model in money market (Engle 1982, and Bollerslev 1986). It was shown that GARCH model cannot analyze hyperbolic memory in conditional volatility process, proposed by Baillie et al., FIGARCH (Fractional Integrated GARCH) was applied in the model and this study showed that it is effectively capture both volatility clustering and long memory.

Exchange rate returns, as a financial time series, is well described by two so-called stylized facts: excess kurtosis of the unconditional distribution, and volatility clustering. The GARCH framework has been suggested and extensively analyzed as an alternative assumption on the conditional distribution. The best known examples are probably the use of the Student's t-distribution in Bollerslev (1987) and of the Generalized Error distribution in Nelson (1991). Both of these distributions can potentially account for the excess kurtosis often found in the standardized residuals from the GARCH models, and a general finding in these studies is that financial return distributions indeed do have fatter tails than the Gaussian distribution. Barndorff-Nielsen (1997) Anderson (2001) Jenson and Lunde(2001) proposed the Normal Inverse Gaussian (NIG) distribution,as a Normal variance-mean mixtures with an Inverse Gaussian mixing distribution, represented by Anderson (2001) Forsberg and Bollerslev (2002), GARCH model with NIG error distribution fits better in the tails of the distribution than the *t* distribution. Moreover, Jensen and Lunde (2001) added that NIG error distributed fits better not only at the tails of daily stock index returns but also at the center of the distribution. Forsberg and Bollerslev represented the relationship of NIG distribution for conditional returns by linking the conditional variance with conditional realized volatility which can be solved from the summation of high-frequency intraday returns.

This study focuses on the conditional heteroscedastic or volatility clustering on the exchange rate of US Dollar, Japan Yen and Thai Baht. The conditional heteroscedastic estimation relies on the long memory in conditional second moment (Baillie, 1989), (Hsieh, 1989) and (Baillie et al., 1996). The empirical analysis is interstingly involved with the use of NIG distribution by analyzing two daily exchange rate returns in order to compare the performance between GARCH and FIGARCH models with different error distribution which are Normal, Student's t and NIG distribution by studying volatility clustering and hyperbolic decay or long range persistence in volatility process.

2. Theory and Methods.

2.1. GARCH and FIGARCH. GARCH (p,q) model came from conditional variance process as:

$$\sigma_t^2 = \omega + \sum_{j=1}^{q} \alpha_j \mu_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{1}$$

From Baillie et al. (1996) study, FIGARCH (p,d,q) model considered the hyperbolic decay in volatility process which is defined as:

$$\phi(L)(1-L)^d u_t^2 = \omega + [1-\beta(L)]\upsilon_t \tag{2}$$

where $\phi(L) = [1-\beta(L)-\alpha(L)]$ and all the roots of $\phi(L)$ and $[1-\beta(L)]$ lie outside the unit circle, $\upsilon_t = u_t^2 - \sigma_t^2$ and $0 < d < 1$. For $0 < d < 1, \phi(L)$ is an infinite order polynomial. As it is evident from (2), FIGARCH model nests GARCH and integrated GARCH (IGARCH) models in the sense that when d = 0 FIGARCH model reduces to GARCH model while for d = 1, it becomes an IGARCH model. Equation (2) can be rearranged for FIGARCH model as:

$$[1-\beta(L)]\sigma_t^2 = \omega + [1-\beta(L)-\phi(L)(1-L)^d]u_t^2 \tag{3}$$

From this equation, the conditional variance of u_t or infinite ARCH which is the representative of FIGARCH process can be written as:

$$\sigma_t^2 = \omega/(1-\beta(1)) + \left[1-\frac{\phi(L)}{1-\beta(L)}(1-L)^d\right]u_t^2 = \frac{\omega}{1-\beta(1)} + \lambda(L)u_t^2 \tag{4}$$

where $\lambda(L) = \lambda_1 L + \lambda_2 L^2 + ...$ and the conditional variance is positive value for all t and the coefficient of infinite ARCH in equation (4) must be nonnegative value which is $\lambda_j \geq 0$ for j=1,2,…. For $0 < d \leq 1, \lambda(1) = 0$ the second moment of the unconditional distribution of u_t is infinite which is similar with IGARCH process, FIGARCH process is not covariance stationary. The *FIGARCH* (*p; d; q*) process is strictly stationary and ergodic for 0 < *d* · 1, but not square integrable (Zaffaroni, 2000).

The outstanding feature of GARCH and FIGARCH model which its volatility relates to the size of the variations in the conditional volatility is amplitude. This feature referred that the amplitude in ARCH can define the size of variation of conditional variance and it can be computed from the sum of coefficient in ARCH (∞) equation represented in GARCH and FIGARCH model. A necessary and sufficient condition for covariance stationary is the condition that the amplitude is less than unity. As shown in Davidson (2004), for the GARCH (1,1) model amplitude will be given by $\alpha_1/(1-\beta_1)$ and *GARCH*(1,1) model is covariance stationary provided that $\alpha + \beta < 1$. However, for *FIGARCH* (1,*d*,1) model the amplitude is restricted to be unity and then FIGARCH model is not covariance stationary.

Another important and useful feature of GARCH and FIGARCH models, is the memory of the conditional volatility process (Baillie et al. (1996), Zaffaroni (2000) and Davidson (2004)) which memory indicates the period of shock that vanish the volatility. In GARCH and FIGARCH model, memory can be classified into two level; geometric (short) memory and hyperbolic (long) memory. Stationary GARCH model has mostly short memory property which means shock has short-lived effects in conditional volatility. For FIGARCH model, FIGARCH process represents the gradually hyperbolic rate of decay for autocorrelation of u_t^2 which is the long memory characteristic.

As mentioned in Davidson(2004) study, the length of memory of conditional volatility is a function of parameter in the model, moreover, geometric memory can be measured by $\theta_i = O(\rho^{-i})$ while hyperbolic-long- memory can be measured by $\theta_i = O(\rho^{-1-\delta})$ where ρ and δ are given by the parameters of the underlying GARCH ,IGARCH and FIGARCH models respectively. In addition, length of the memory varies inversely with ρ and δ, the distinct example is in GARCH(1,1) model which is short memory. From $\rho = \frac{1}{\beta}$, it is found that the more β_1, the shorter length of memory in GARCH(1,1) model is. In contrasts with FIGARCH (p,d,q), the memory of the process is increasing as *d* approaches to zero rather than unity.

Davidson also pointed out that the length of persistence of shock to the conditional volatility is inversely related to the hyperbolic decay parameter d in FIGARCH (p,d,q) model.

In FIGARCH (p,d,q), length of the memory in the process increases when the value d approaches to zero. When d becomes unity, which denotes that FIGARCH model reduces to an IGARCH model, the memory of the process jumps to short memory and when d decreases and approaches to zero, (but not becomes exactly zero), the length of the process increases. When $d = 0$, then the process becomes short memory GARCH model. Hence, the characteristic of the FIGARCH process linked between GARCH and IGARCH may be inexact. According to the FIGARCH model with NIG error distributed, its usage is suitably proper in both hyperbolic memory and exchange rate return. The estimation of memory parameter d is useful when the data involves with length of the memory in conditional volatility of the return of exchange rate and other financial data.

2.2. Normal Inverse Gaussian Distribution. Normal Inverse Gaussian error distribution was from speculative model in the study of Barndoff-Nielson (1997), Anderson (2001), Jenson and Lunde (2001) using GARCH and stochastic relativity. This study shows the density function of NIG distribution with random variable (x) as:

$$NIG(x : a,b,\mu,\delta) = \frac{a}{\pi\delta} exp\left(\sqrt{a^2 - b^2} + b\frac{x-\mu}{\delta}\right) + q\left(\frac{x-\mu}{\delta}\right)^{-1} K_1(aq(\frac{x-\mu}{\delta})) \quad (5)$$

Where $q(y) = \sqrt{1+y^2}$ and $0 \le |b| \le a, \mu \in R, \delta > 0$. Term $K_1(.)$ represents the Bessel function which is modified in the third order and the first index. In this equation, a denotes the slope and b denotes asymmetry of the distribution which turns to be a symmetry distribution when the value is zero, δ is scale parameter and μ is location parameter which is determined as mean when b is zero.

NIG distribution has several interesting properties; the first property is the aggregation which means daily exchange rate data with NIG distribution can be converted to be weekly data as the sum of daily data with the NIG distribution remains. The second property, from Barndorff-Nielsen, the study characterized the semi-heavy tails which is fit in the exchange rate and stock index model. Jensen and Lunde showed, as the third property, NIG distribution is better than student-t and normal distribution as the higher peak than standard normal distribution found. The last property was added by Forsberg and Bollerlev (2002) in the study of the five-minute return from the ECU basket of currencies versus the US dollar, they found that the volatilities constructed from the summation of the high-frequency intraday squared returns conditional on the lagged squared daily returns are Inverse Gaussian (IG) distributed, and hence the implied daily returns should approximately be NIG distributed.

In FIGARCH-NIG model, the time series return data from the exchange rate can be written as;

$$r_t = \mu + \frac{b\sqrt{\gamma}}{a}\sigma_t + z_t\sigma_t,\ for\ t = 1,\dots,T \quad (6)$$

which z_t is zero-mean and unit variance process. From the study of Anderson and Jenson and Lunde defined the z_t to be NIG distributed as;

$$z_t \sim NIG(a, b, -\frac{b\sqrt{\gamma}}{a}, \frac{\gamma^{\frac{3}{2}}}{a})$$

where $\gamma = \sqrt{a^2 - b^2}$ in NIG distributed defined that the conditional distribution of returns will be NIG as well;

$$r_t \mid \Omega_{t-1} \sim NIG(a, b, \mu, \frac{\gamma^{\frac{3}{2}}}{a}\sigma_t)$$

where Ω_{t-1} is the information set from the previous day return and $\Omega_{t-1} = \sigma(r_{t-1}, r_{t-1}, r_{t-2}, ...)$ and conditional mean and conditional variance can be defined as

$$E[r_t \mid \Omega_{t-1}] = \mu + \sigma_t \frac{b\sqrt{\gamma}}{a},\ for\ t = 1, ..., T$$

$$Var(r_t \mid \Omega_{t-1}) = \sigma_t^2,\ for\ t = 1, ..., T$$

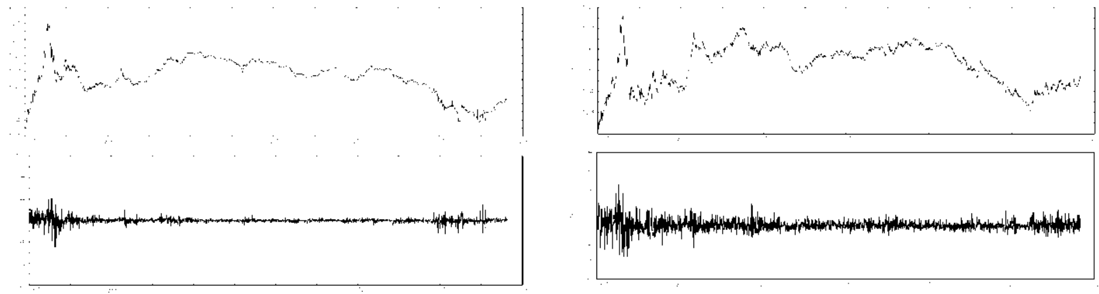
It is given that $u_t = r_t - E(r_t \mid \Omega_{t-1}) = r_t - \sigma_t \frac{b\sqrt{a^2 - b^2}}{a} - \mu$ which is the innovation of return process.

3. Data and Empirical Results.

3.1. Data Description. In this section we study the volatility dynamics in exchange rate markets by utilizing GARCH and FIGARCH model under alternative distributions. The data consists of daily nominal spot exchange rates between the Thai Baht versus US Dollar and Thai Baht versus Japanese Yen. The data is the daily close rate obtained from Reuters 2007 database supported by Faculty of Economics, Chiang Mai University. The sample period is October 21, 1993 to September 12, 2008, totally 3,879 observations. Therefore, following the standard practice, 3,879 daily exchange rate of returns are constructed as $r_t = \log\left(\frac{S_t}{S_{t-1}}\right)$, where S_t denotes the spot exchange rate at day *t*.

Table 1 reports the summary statistics together with the Box-Pierce (1979) statistics tests for up to 22^{nd}-order serial correlation in the returns and squared returns. The reported results indicate that for all series, daily return average about 0.0% with considerable amount of variation. The sample variance of US Dollar return is smaller than Japanese Yen. Skewness indicates that Thai Baht – US Dollar exchange rate has negative value and Thai Baht – Japanese Yen exchange rate has positive value. The highest kurtosis value is found from US Dollar exchange return.

Figure 1 displays Daily spot exchange rates of returns and squared returns over the sample period. The plot in the figure clearly indicates the occurrence of tranquil and volatile periods.



US Dollar – Thai Baht Japanese Yen – Thai Baht

FIGURE 1. Daily spot exchange rates and returns

Figure 2 and Figure 3 display autocorrelation of transformations of the residuals of US Dollar-Thai Baht and Japanese Yen- Thai Baht respectively. As shown in the autocorrelation graph, both daily return series are uncorrelated through time. Q(22) values indicates some persistence in the return for both Dollar-Thai Baht and Japanese Yen- Thai Baht. This is also supported by the Box-Pierce statistic for up to 22nd-order serial correlation in daily squared returns.

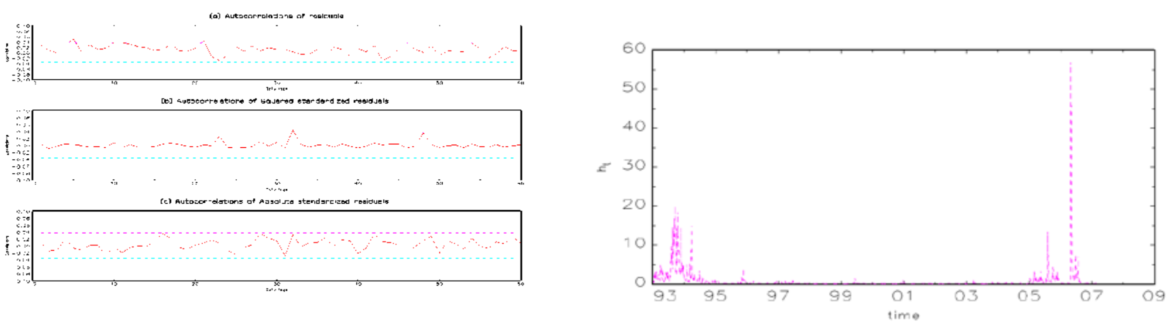


FIGURE 2. Autocorrelation of Transformations of the Residuals of US Dollar-Thai Baht and Estimated Conditional Variance of US Dollar-Thai Baht

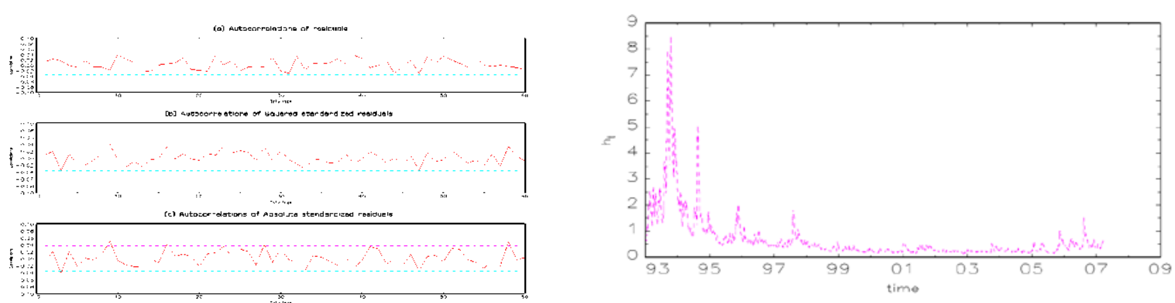


FIGURE 3. Autocorrelation of Transformations of the Residuals of Japanese Yen-Thai Baht and Estimated Conditional Variance of Japanese Yen-Thai Baht

US Dollar – Thai Baht Japanese Yen – Thai Baht

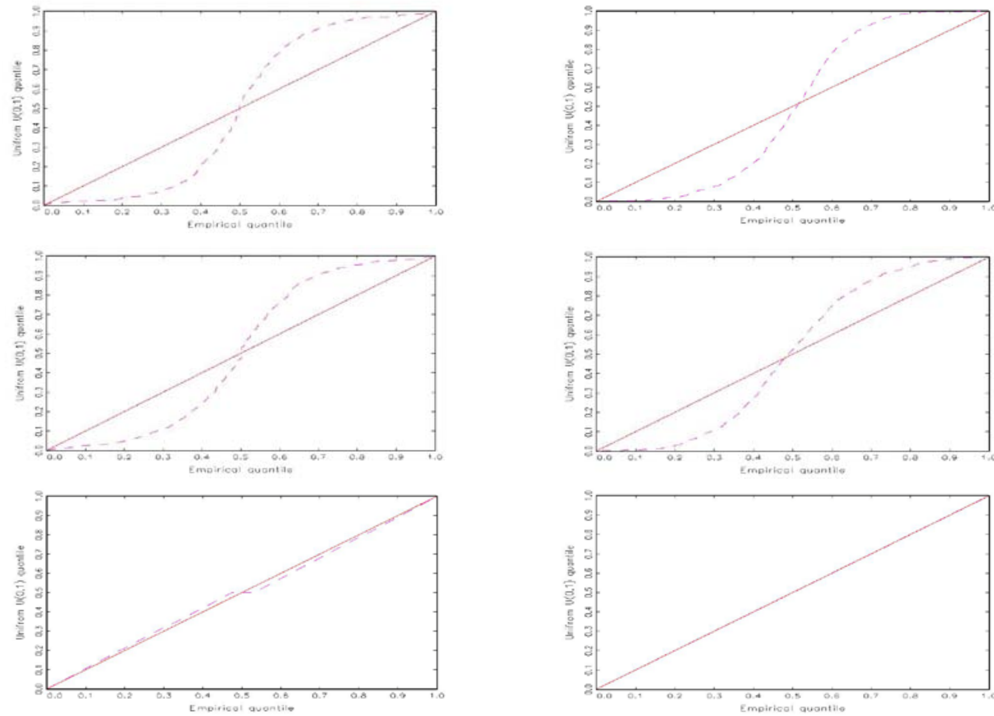


FIGURE 4. Q plot for the PITs from FIGARCH models with normal, student's t and NIG errors

US Dollar – Thai Baht Japanese Yen – Thai Baht

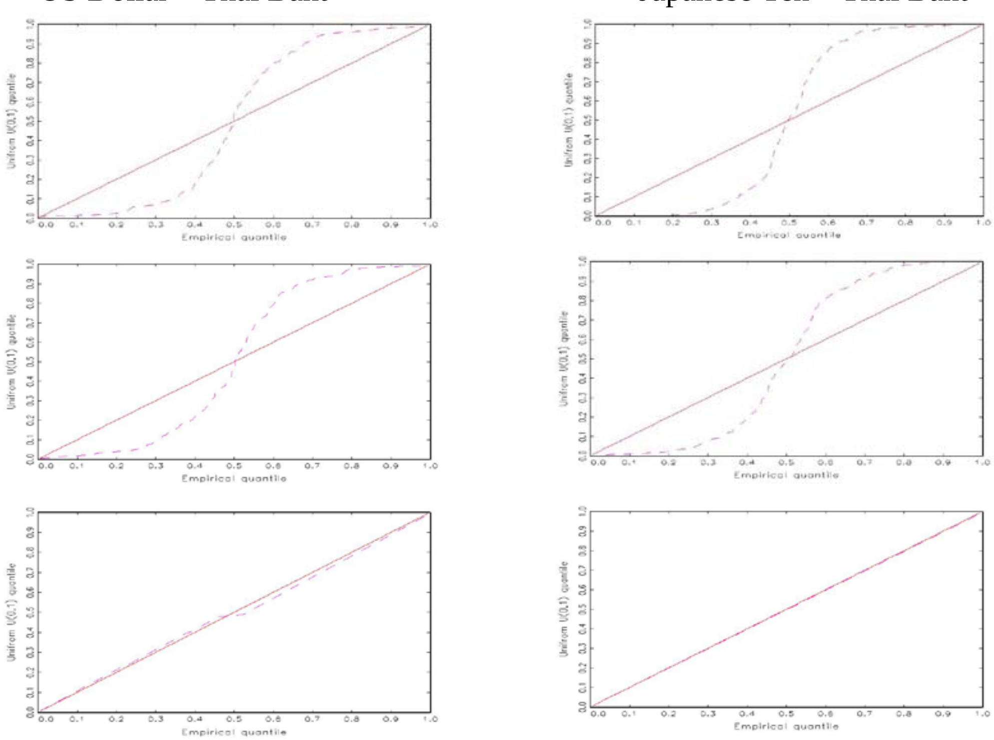


FIGURE 5. QQ plot for the PITs from FIGARCH models with normal, student's t and NIG errors

3.2. Distribution Estimations. Parameter estimates, standard error and summary diagnostic statistics for GARCH and FIGARCH models with Normal, student's t and NIG errors from Quasi Maximum Likelihood method are presented in Table 3 and 4. A comparison of estimated GARCH and FIGARCH models with any given distribution assumption, in term of lowering the kurtosis in residuals and Box-Pierce statistics computed from standardized residuals and squared residuals and likelihood values generally favors a FIGARCH specification for the conditional volatility of daily exchange rate returns. For both exchange rate, the estimates of hyperbolic decay parameter, *d* is significant and greater than 0 but less than unity and are in the range of about 0.3 – 0.7 from FIGARCH model.

Figure 4 and 5 show that the FIGACH (1,d,1) model with NIG distribution fits the Thai Baht – Japanese Yen quite symmetrically. A statistically significant estimate of parameter *b*, with positive values, indicates presence of symmetry in the distribution of Thai exchange rate daily return. Furthermore, the estimated values for *a* indicate that the Thai Baht – Japanese Yen returns have the most peaked distribution.

4. Conclusion. In this paper, we proposed the FIGARCH – NIG model to study the hyperbolic decay, time varying dynamic in conditional volatility and peakedness, asymmetry and fat tailed distribution of daily Thai Baht – US Dollar and Thai Baht – Japanese Yen rate returns.

From this research, the estimated GARCH and FIGARCH models, with normal, *t* and the NIG distributions compared in terms of sample fit, distinctly found hyperbolic memory in conditional volatility and asymmetric distribution of US Dollar daily exchange return but symmetric distribution of Japanese Yen daily exchange rate return, both were found with FIGARCH model analysis. However, how to extend the model and choose an appropriate estimation is still the open question.

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Appendix.

TABLE 1. Summary statistics for daily exchange rate returns

	mean	med	min	max	var	skew	kurt
US Dollar	0.007	0.000	-11.737	11.072	0.668	-0.526	44.027
Japanese Yen	0.009	-0.050	-5.602	7.203	0.678	0.080	11.526

TABLE 2. Quantile Predictions-Value-at-Risk for FIGARCH models with Normal, student's t, and NIG errors

1-day ahead

Nominal		1	5	10	90	95	99
USD-THB	FG-N	2.805	23.666	34.348	67.699	72.234	94.690
	FG-t	4.299	24.878	30.833	66.391	77.183	85.612
	FG-NIG	0.937	4.688	9.375	90.625	95.313	99.063
JPY-THB	FG-N	23.648	31.016	37.064	61.966	66.797	76.965
	FG-t	10.276	27.087	33.320	67.729	74.075	82.006
	FG-NIG	1.000	5.000	10.000	90.000	95.000	99.000

TABLE 3. GARCH and FIGARCH Models: Daily US Dollar - Thai Baht Returns

	G - N	G - t	G - NIG	G - NIGb	FG - N	FG - t	FG - NIG	FG - NIGb
μ	0.0010	-0.0054	-0.0050	-0.0128	-0.0023	-0.0069	-0.0063	-0.0138
	0.0055	0.0054	0.0052	0.0071	0.0057	0.0054	0.0052	0.0071
θ	-0.0057	-0.0033	-0.0055	-0.0053	0.0001	-0.0016	-0.0052	-0.0057
	0.0167	0.0176	0.0174	0.0174	0.0196	0.0189	0.0187	0.0187
d	-	-	-	-	0.7091	0.6880	0.7000	0.7013
	-	-	-	-	0.0211	0.0602	0.0610	0.0614
ω	0.0042	0.0063	0.0053	0.0053	0.0084	0.0118	0.0097	0.0097
	0.0003	0.0011	0.0010	0.0010	0.0007	0.0031	0.0026	0.0026
β	0.8489	0.8147	0.8177	0.8178	0.6098	0.5935	0.6044	0.6061
	0.0039	0.0146	0.0145	0.0145	0.0228	0.0844	0.0828	0.0832
α	0.1501	0.8143	0.1813	0.1812	-	-	-	-
	0.0039	0.0146	0.0145	0.0145	-	-	-	-
φ	-	-	-	-	0.0450	0.1936	0.1916	0.1932
	-	-	-	-	0.0175	0.0763	0.0760	0.0768
υ	-	3.6365	-	-	-	3.5755	-	-
	-	0.1855	-	-	-	0.1797	-	-
a	-	-	0.6634	0.6693	-	-	0.6504	0.6556
	-	-	0.0642	0.0660	-	-	0.0625	0.0643
b	-	-	-	0.0322	-	-	-	0.0315
	-	-	-	0.0201	-	-	-	0.0203
ll	-2024.33	-1643.68	-1648.05	-1646.88	-1999.00	-1631.90	-1636.77	-1635.64
AIC	4058.67	3299.36	3308.09	3307.77	4010.01	3277.80	3287.54	3287.28
SIC	4088.56	3335.23	3343.96	3349.61	4045.87	3319.64	3329.38	3335.11
m3	1.7810	1.7100	1.6650	0.1760	1.4760	1.7740	1.7010	1.6800
m4	33.9500	31.6550	30.7980	7.5290	28.2210	32.4970	31.1010	31.4310
Q	30.5160	29.2964	30.3532	29.0382	30.3092	29.3502	30.7616	29.3774
Q^2	2.3755	2.3411	2.3685	2.3411	2.9288	1.4705	1.5583	1.5028
$Q_{pit}(1)$	0.0975	0.0975	0.0936	0.0958	1.2980	1.7650	1.8681	1.8774
$Q_{pit}(5)$	0.1137	0.1222	0.1145	0.1168	1.8049	2.9741	3.2753	3.3252
$Q_{pit}(10)$	0.1339	0.1528	0.1406	0.1429	2.2132	3.9614	4.4619	4.5533
$W_{d=1}$	-	-	-	-	189.7200	26.8720	24.2020	23.6770
α+β=1	-	∞	∞	∞	126.7230	2.2040	2.1230	2.0170

TABLE 4. GARCH and FIGARCH Models: Daily Japanese Yen - Thai Baht Returns

	G - N	G - t	G - NIG	G - NIGb	FG - N	FG - t	FG - NIG	FG - NIGb
μ	-0.0075	-0.017	-0.0171	-0.1084	-0.0073	-0.017	-0.0171	-0.1032
	0.0104	0.0098	0.0098	0.0208	0.0104	0.0098	0.0098	0.0212
θ	-0.0353	-0.0434	-0.0425	-0.0408	-0.0309	-0.041	-0.0399	-0.0458
	0.0198	0.0189	0.0188	0.0188	0.021	0.0191	0.0192	0.0192
d	-	-	-	-	0.3879	0.416	0.4055	0.3883
	-	-	-	-	0.0426	0.0711	0.0691	0.0649
ω	0.0039	0.0045	0.0044	0.0044	0.0138	0.0157	0.015	0.0133
	0.0008	0.0014	0.0014	0.0014	0.0028	0.0059	0.0057	0.0059
β	0.9346	0.9326	0.9329	0.9299	0.6234	0.6011	0.5993	0.5751
	0.0056	0.009	0.009	0.0093	0.0239	0.0765	0.0799	0.0775
α	0.0586	0.0591	0.0585	0.0598	-	-	-	-
	0.0053	0.0086	0.0085	0.0085	-	-	-	-
φ	-	-	-	-	0.3266	0.2459	0.258	0.2542
	-	-	-	-	0.0239	0.0593	0.0604	0.0628
υ	-	7.0444	-	-	-	6.928	-	-
	-	0.8968	-	-	-	0.8727	-	-
a	-	-	2.1932	2.3153	-	-	2.1787	2.2445
	-	-	0.3588	0.0231	-	-	0.3666	0.3733
b	-	-	-	0.2968	-	-	-	0.2767
	-	-	-	0.0607	-	-	-	0.0723
ll	-2931.33	-2875.59	-2875.98	-2864.96	-2925.77	-2873.83	-2874.33	-2864.36
AIC	5872.67	5763.19	5763.95	5743.92	5863.54	5761.67	5762.66	5744.71
SIC	5902.56	5799.05	5799.82	5785.77	5899.41	5803.51	5804.51	5792.53
m3	0.4400	0.4920	0.4920	0.2540	0.4060	0.4710	0.4680	0.0720
m4	4.6020	4.6370	4.6360	4.3920	4.4600	4.6040	4.5840	4.3700
Q	18.2240	18.6270	18.5526	18.9237	17.6463	18.0044	17.9247	18.2076
Q^2	20.4413	20.6412	20.6699	17.2444	20.3235	20.4229	20.2800	17.3481
$Q_{pit}(1)$	0.5529	0.5542	0.5509	0.5849	2.3569	1.8637	1.8887	1.6666
$Q_{pit}(5)$	0.5533	0.5538	0.5498	0.5787	4.9661	3.3460	3.3951	2.6832
$Q_{pit}(10)$	0.5539	0.5533	0.5484	0.5712	7.5209	4.5929	4.6675	3.4992
$W_{d=1}$	-	-	-	-	206.3470	67.4720	74.1140	88.7200
$\alpha + \beta = 1$	8.9670	4.1680	4.7350	6.5500	∞	1.7850	1.5380	1.9360

	G - N	G - t	G - NIG	G - NIGb	FG - N	FG - t	FG - NIG	FG - NIGb
d	-	-	-	-	0.7091	0.6880	0.7000	0.7013
	-	-	-	-	0.0211	0.0602	0.0610	0.0614
ω	0.0042	0.0063	0.0053	0.0053	0.0084	0.0118	0.0097	0.0097
	0.0003	0.0011	0.0010	0.0010	0.0007	0.0031	0.0026	0.0026
β	0.8489	0.8147	0.8177	0.8178	0.6098	0.5935	0.6044	0.6061
	0.0039	0.0146	0.0145	0.0145	0.0228	0.0844	0.0828	0.0832
α	0.1501	0.8143	0.1813	0.1812	-	-	-	-
	0.0039	0.0146	0.0145	0.0145	-	-	-	-
φ	-	-	-	-	0.0450	0.1936	0.1916	0.1932
	-	-	-	-	0.0175	0.0763	0.0760	0.0768
a	-	-	0.6634	0.6693	-	-	0.6504	0.6556
	-	-	0.0642	0.0660	-	-	0.0625	0.0643
b	-	-	-	0.0322	-	-	-	0.0315
	-	-	-	0.0201	-	-	-	0.0203
AIC	4058.67	3299.36	3308.09	3307.77	4010.01	3277.80	3287.54	3287.28
SIC	4088.56	3335.23	3343.96	3349.61	4045.87	3319.64	3329.38	3335.11

	G - N	G - t	G - NIG	G - NIGb	FG - N	FG - t	FG - NIG	FG - NIGb
d	-	-	-	-	0.3879	0.416	0.4055	0.3883
	-	-	-	-	0.0426	0.0711	0.0691	0.0649
ω	0.0039	0.0045	0.0044	0.0044	0.0138	0.0157	0.015	0.0133
	0.0008	0.0014	0.0014	0.0014	0.0028	0.0059	0.0057	0.0059
β	0.9346	0.9326	0.9329	0.9299	0.6234	0.6011	0.5993	0.5751
	0.0056	0.009	0.009	0.0093	0.0239	0.0765	0.0799	0.0775
α	0.0586	0.0591	0.0585	0.0598	-	-	-	-
	0.0053	0.0086	0.0085	0.0085	-	-	-	-
φ	-	-	-	-	0.3266	0.2459	0.258	0.2542
	-	-	-	-	0.0239	0.0593	0.0604	0.0628
a	-	-	2.1932	2.3153	-	-	2.1787	2.2445
	-	-	0.3588	0.0231	-	-	0.3666	0.3733
b	-	-	-	0.2968	-	-	-	0.2767
	-	-	-	0.0607	-	-	-	0.0723
AIC	5872.67	5763.19	5763.95	5743.92	5863.54	5761.67	5762.66	5744.71
SIC	5902.56	5799.05	5799.82	5785.77	5899.41	5803.51	5804.51	5792.53